SAARI TRIANGLES AND KOCH SNOWFLAKES : VOTING EXAMPLES WITH PREFRACTAL IMAGES

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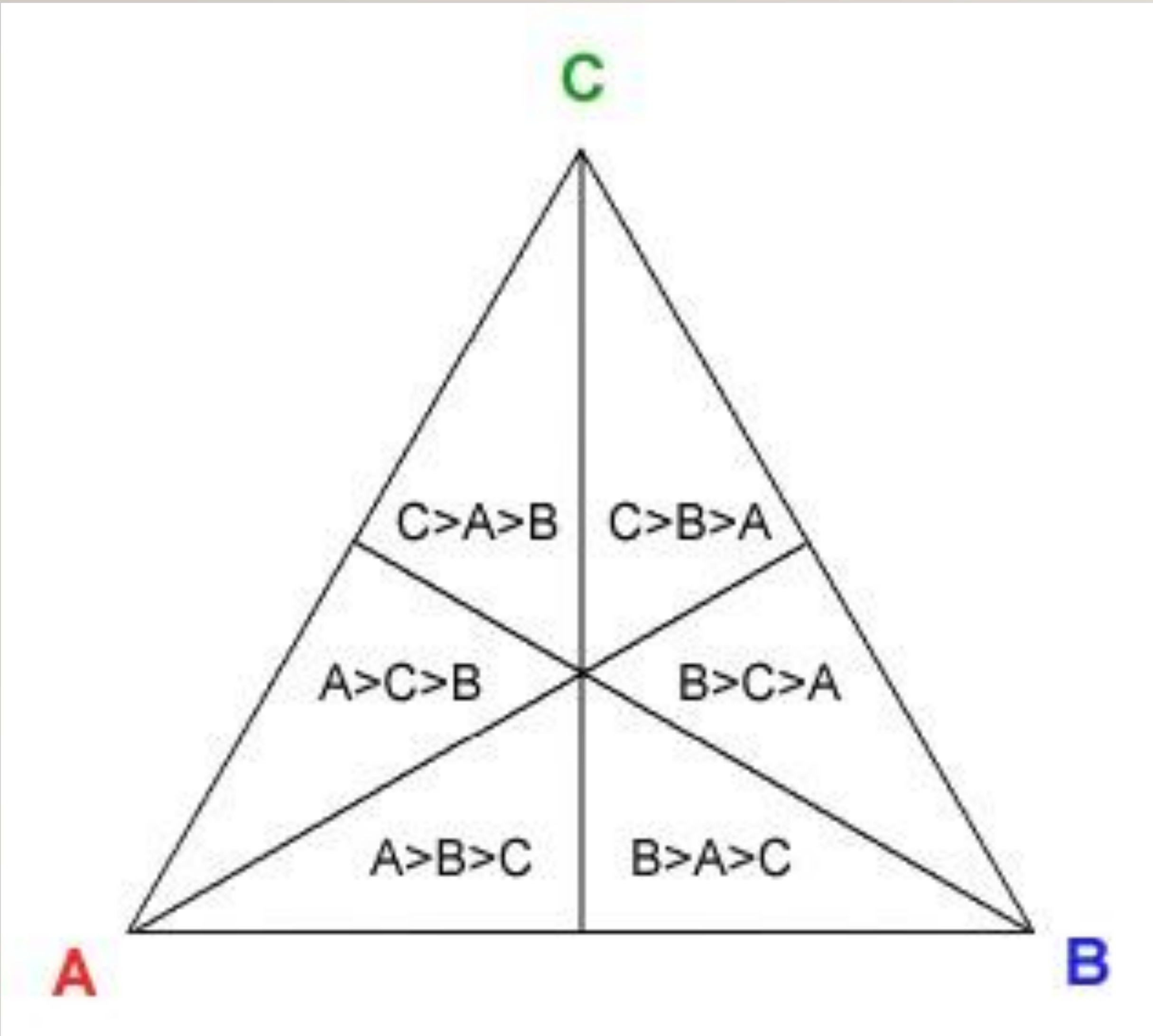
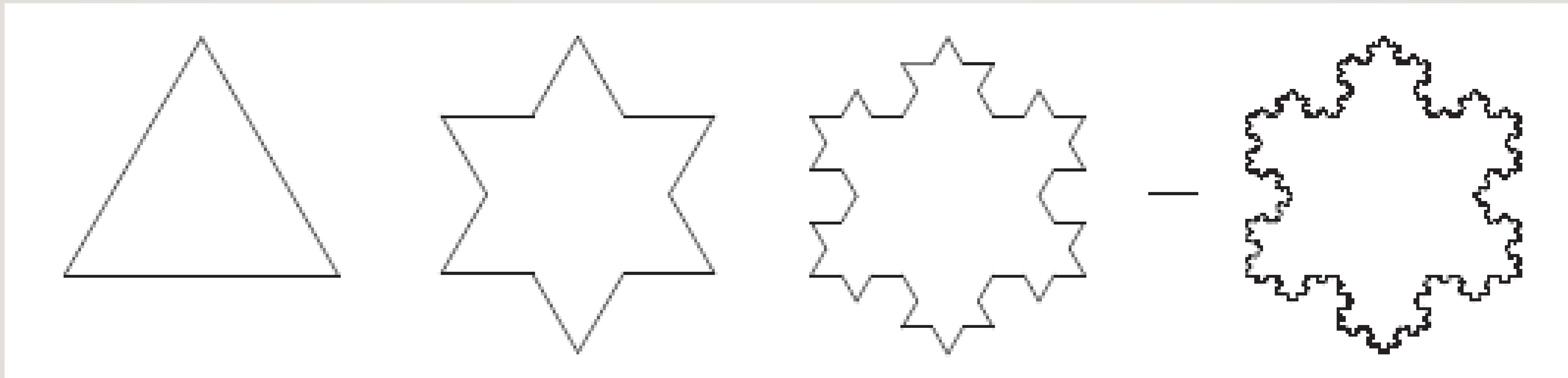
VOTING PROCEDURES DESCRIBE THE MANNER IN WHICH THE PREFERENCES OF INDIVIDUALS ARE COMBINED TO PRODUCE A COLLECTIVE DECISION - AN ELECTION OR DECISION OUTCOME NOT NECESSARILY REVEALS THE TRUE PREFERENCES OF THE VOTERS BUT MOREOVER THE CHOICE OF AN ELECTION RULE.

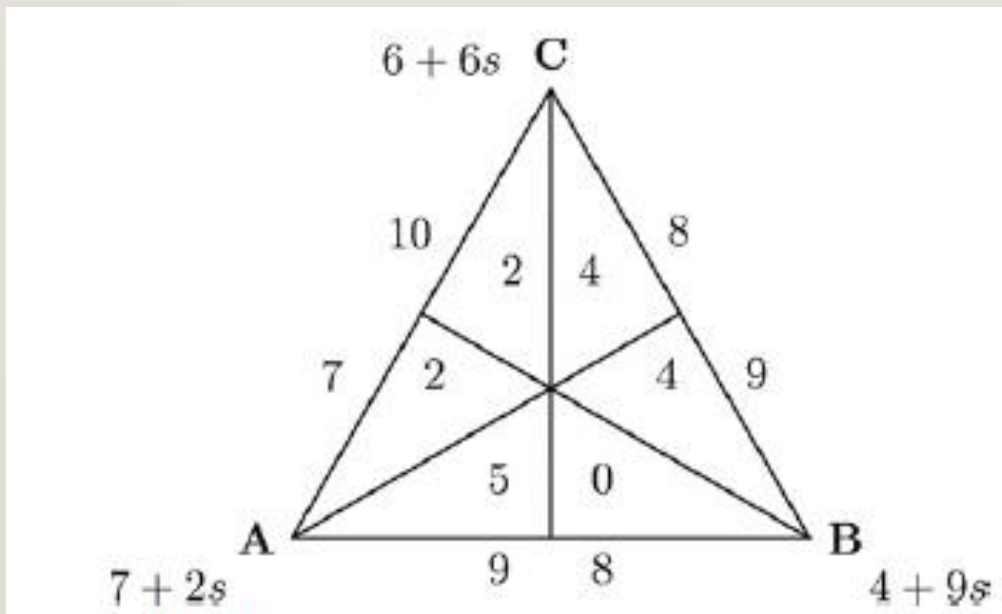
THE SAARI REPRESENTATION TRIANGLE - A GEOMETRIC PROFILE REPRESENTATION (I.E. AN EQUILATERAL TRIANGLE SIMPLEX)

A POSITIONAL ELECTION WITH THE THREE CANDIDATES A, B, AND C IS DEFINED BY THE (NORMALIZED) VOTING VECTOR W(S)=W(1),W(2),W(3)=(1,S,0), WHERE S , 0≤S≤1, IS A SPECIFIED WEIGHT FOR A SECOND-RANKED ALTERNATIVE (I.E. CANDIDATE). **S=0**, THE POSITIONAL RULE REDUCES TO THE PLURALITY METHOD; **W(PL)=(1,0,0).S=1**,THE ANTIPLURALITY RULE WHICH GIVES THE RESULT **W(APL)=(1,1,0) (=I.E. AGAINST THE THIRD-PLACE CANDIDATE).S=½** GIVES THE BORDA COUNT. **W(BC)=(2,1,0).**

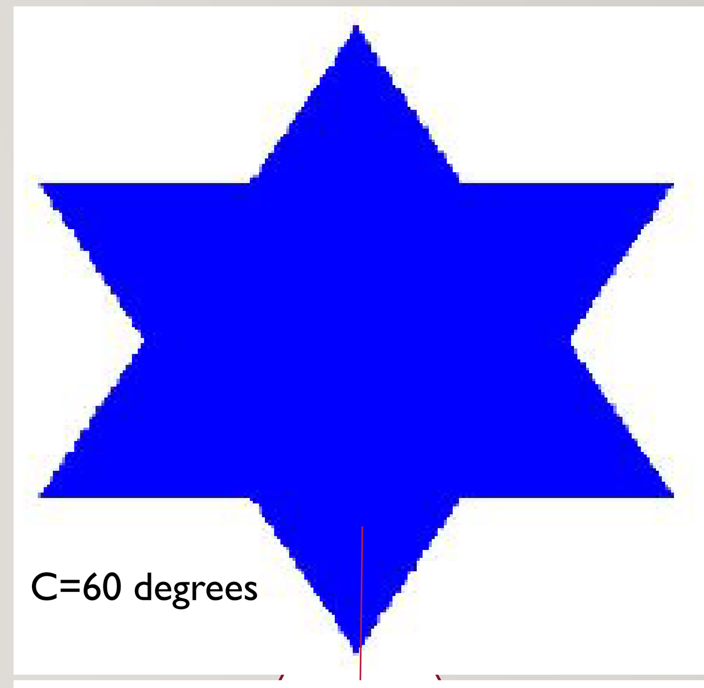
THE KOCH SNOWFLAKE - THE KOCH SNOWAKE CURVE (KS) AND ITS PREFRACTAL APPROXIMATIONS KS(N), FOR N = 0; 1; 2; _ _ _ . KS IS A FRACTAL, NOWHERE DIFFERENTIABLE AND CLOSED CURVE, OF INFINITE LENGTH, WITH A FINITE AREA.IT IS THE UNION OF THREE SELF-SIMILAR SETS, EACH AN ISOMETRIC COPY OF THE CLASSIC KOCH CURVE. THE INITIATOR OF THE KOCH SNOWFLAKE OR THE TRIADIC KOCH CURVE IS AN EQUILATERAL (E.G. SAARI ?) TRIANGLE – GENERATORS ARE SELF-SIMILAR OBJECTS OBTAINED BY THE RECURSIVE ITERATION RULE. FOR THE STANDARD KOCH CURVE, INDENTATION ANGLE (*Θ*) =60◦, *N* = 4, *S* = 3, AND THE SELF-SIMILARITY DIMENSION *D= - LOG N/ LOG(1/S)* = 1.261 (N=NUMBER OF COPIES,S=SCALING FACTOR). IF THE INDENTATION ANGLE IS MADE VARIABLE, THE SCALING FACTOR BECOMES FUNCTION OF *Θ* . 1/S=1/2(1+COS *Θ*). GEOMETRIES WITH VARYING FRACTAL DIMENSIONS **ARE OBTAINED** (SEE THE BLUE FIGURES!)







(a) Profile.



C=60 degrees

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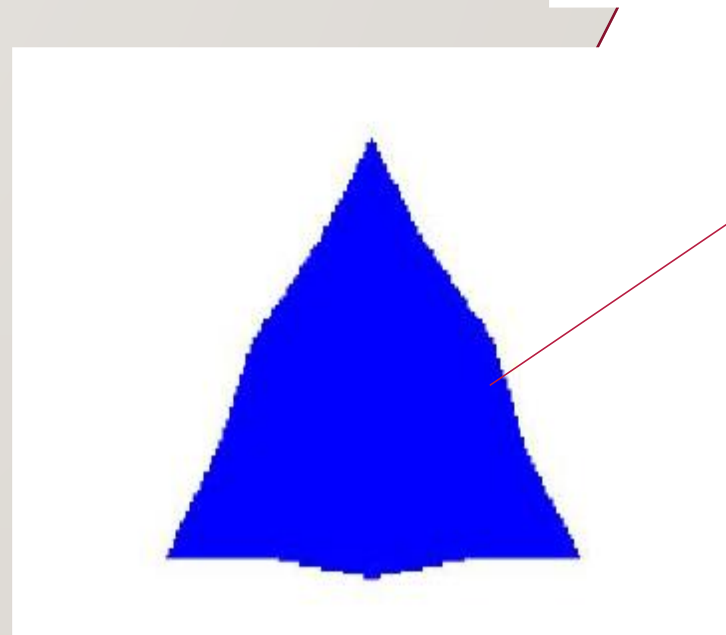
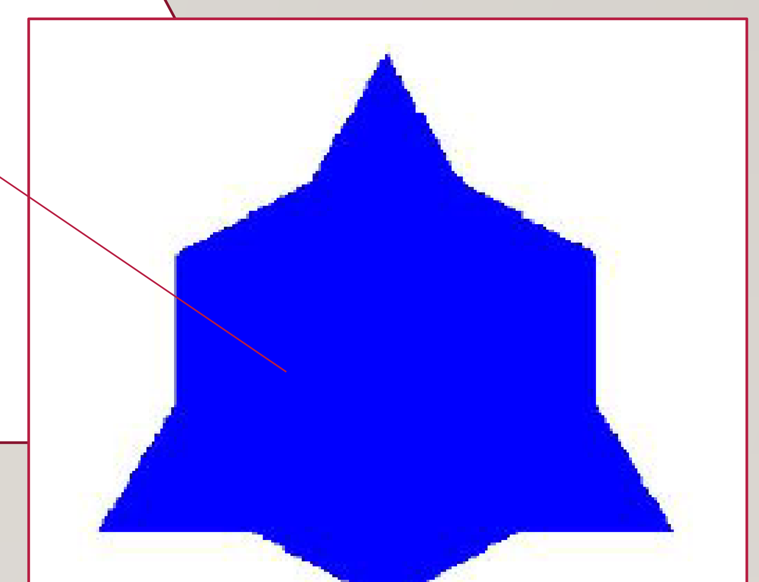


A=10 degrees



B=30 degrees

AN EXAMPLE OF A PREFERENCE PROFILE			
NUMBER OF VOTES	RANKING	NUMBER OF VOTES	RANKING
5	A>B>C	4	C>B>A
2	A>C>B	2	C>A>B
4	B>C>A		

WITH THE PLURALITY RULE ("VOTE FOR ONE")" A WINS >>> THE INDENTATION ANGLE 10 DEGREES PREVAILS : SEE THE BLUE FIGURES!

WITH THE ANTIPLURAITY RULE ("VOTE FOR TWO") B WINS >>> THE INDENTATION ANGLE 30 DEGREES PREVAILS: AS ABOVE!

WITH THE BORDA COUNT (2,1,0) C WINS >>> SEE THE BLUE FIGURES!

IN THE FUTURE: TRUE MULTIFRACTAL SOLUTIONS (?) – CONCURRENTLY ADDING UP NUMBER OF VOTES AND KOCH SNOWFLAKE ITERATIONS TOWARDS THE INFINITY - PACED WITH RANDOMIZING THE PARAMETER S OF THE POSITIONAL RULE RECURSIVELY???

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