SAARI TRIANGLES AND KOCH SNOWFLAKES: VOTING EXAMPLES WITH PREFRACTAL IMAGES

FREE-LANCE RES., JAACKO HAKULA, WITH CONNECTIONS TO THE UNIVERSITIES OF OULU AND TURKU, FINLAND

VOTING PROCEDURES DESCRIBE THE MANNER IN WHICH THE PREFERENCES OF INDIVIDUALS ARE COMBINED TO PRODUCE A COLLECTIVE DECISION - AN ELECTION OR DECISION OUTCOME NOT NECESSARILY REVEALS THE TRUE PREFERENCES OF THE VOTERS BUT MOREOVER THE CHOICE OF AN ELECTION RULE.

THE SAARI REPRESENTATION TRIANGLE - A GEOMETRIC PROFILE REPRESENTATION (I.E. AN EQUILATERAL TRIANGLE SIMPLEX)

A POSITIONAL ELECTION WITH THE THREE CANDIDATES A, B, AND C IS DEFINED BY THE (NORMALIZED) VOTING VECTOR W(S)=W(1),W(2),W(3)=(1,S,0), WHERE S , 0≤S≤1, IS A SPECIFIED WEIGHT FOR A SECOND-RANKED ALTERNATIVE (I.E. CANDIDATE). S=0, THE POSITIONAL RULE REDUCES TO THE PLURALITY METHOD: W(PL)=(1,0,0). THE ANTIPLURALITY RULE WHICH GIVES THE RESULT W(APL)=(1,1,0) (I.E. AGAINST THE THIRD-PLACE CANDIDATE). S=½ GIVES THE BORDA COUNT. W(BC)=(2,1,0).

THE KOCH SNOWFLAKE - THE KOCH SNOWFLAKE CURVE (KS) AND ITS PREFRACTAL APPROXIMATIONS KS(N), FOR N = 0; 1; 2; _ _ _ . KS IS A FRACTAL, NOWHERE DIFFERENTIABLE AND CLOSED CURVE, OF INFINITE LENGTH, WITH A FINITE AREA. IT IS THE UNION OF THREE SELF-SIMILAR SETS, EACH AN ISOMETRIC COPY OF THE CLASSIC KOCH CURVE. THE INITIATOR OF THE KOCH SNOWFLAKE OR THE TRIADIC KOCH CURVE IS AN EQUILATERAL (E.G. SAARI?) TRIANGLE – GENERATORS ARE SELF-SIMILAR OBJECTS OBTAINED BY THE RECURSIVE ITERATION RULE. FOR THE STANDARD KOCH CURVE, INDENTATION ANGLE (Θ) = 60°, N = 4, S = 3, SIMILARITY DIMENSION D = - LOG N / LOG(1/S) = 1.261 (N=NUMBER OF COPIES, S=SCALING FACTOR). IF THE INDENTATION ANGLE IS MADE VARIABLE, THE SCALING FACTOR BECOMES 1/S=1/2(1+COSΘ).

AN EXAMPLE OF A PREFERENCE PROFILE

<table>
<thead>
<tr>
<th>NUMBER OF VOTES</th>
<th>RANKING</th>
<th>NUMBER OF VOTES</th>
<th>RANKING</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>A&gt;B&gt;C</td>
<td>4</td>
<td>C&gt;B&gt;A</td>
</tr>
<tr>
<td>2</td>
<td>A&gt;C&gt;B</td>
<td>2</td>
<td>C&gt;B&gt;A</td>
</tr>
<tr>
<td>4</td>
<td>B&gt;A&gt;C</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

WITH THE PLURALITY RULE ("VOTE FOR ONE") A WINS >>> THE INDENTATION ANGLE 10 DEGREES PREVAILS : SEE THE BLUE FIGURES!

WITH THE ANTIPLURALITY RULE ("VOTE FOR TWO") B WINS >>> THE INDENTATION ANGLE 30 DEGREES PREVAILS: AS ABOVE!

WITH THE BORDA COUNT (2,1,0) C WINS >>> SEE THE BLUE FIGURES!

IN THE FUTURE: TRUE MULTIFRACTAL SOLUTIONS (?) – CONCURRENTLY ADDING UP NUMBER OF VOTES AND KOCH SNOWFLAKE ITERATIONS TOWARDS THE INFINITY - FACED WITH RANDOMIZING THE PARAMETER S OF THE POSITIONAL RULE RECURSIVELY???

JAACKO HAKULA: e-mail address jaakko.hakula@dmainternet.net