Relation between sensitive systems, topological entropy and Baire set in MDS

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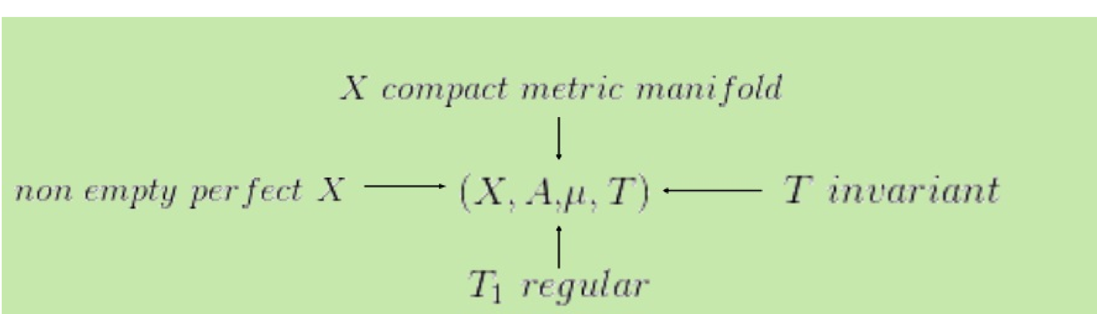
Abstract

In this article we going to use a Measurable dynamical system of the form (X,B,μ,T,S) for do some analysis about the connection between the dynamics that are present in C(X) and the strong relation between ω-scrambled set and topological entropy for homeomorphics maps with horse-shoe property.

Introduction

- In many works about chaos, the Researchers had searched answers thrown those behaviours of the trajectories seeing in some systems and define chaos as maps in sensitive systems. Other groups of work prefer to see a chaotic system from an uncountable set that has positive Lebesgue measure and thus obtain results from the point of view of Ergodic theory and Topology. In both cases, the problem is about how we can establish a relation between the projection seeing in some special behaviours found in a countable number of orbits and the analytic form in which we can talk about Strong laws that are present in sensitive systems.
- In this article, we focus our attention in chaotic behaviour, for a Dynamical System (X,T), where X contain a uncountable subset known as scrambled set and thrown the relation between some sensitive subsystem on Devaney maps and the continuity state of recurrent limits sets, we prove that a chaotic system contain a collection of characteristic linked to the existence of scattering of the points in chaotic orbits, the exponential grown in which the points start to being far apart and relative importance of each character depending the system in which we are working.
- In the first chapter we going to define a MDS-type1, that is so important in the connection with many type of system and their properties, which we can associated using factor maps, but in this case, we need to study some cases of semi-open homeomorphisms that can be project in the better form the behaviour of other system more complicate to see.
- In second chapter we see some relation between Li-York maps and MDS-type1 for some types of dynamics with Hausdorff metric. As we know for the works of [7] [8], we need to define a relation between scattering system with (n, T, ϵ)-properties, which are joined by the existence of a maximum metric for nearby points with sup norm associated (trivial).
- At the end, we going to study E-systems, defining minimal but not equi-continuous sets and we prove that an E-system isomorphic to an MDS-type 1 had maps with positive topological entropy and Baire category 2, but only for ergodic subsystem with horse-shoe maps included.



MDS-type1

Let (X,B,μ,T,S) a measurable dynamical system (MDS) with S as an action semigroup on $\mathbb{Z}^+$, then this system is a MDS-type1 if the next properties are equivalent:

1. The elements of B are T-invariant under a positive measure μ
2. There exists perfect subsets in X
3. X is T_2
4. X contain compact and metric subsystems

Our objective with the MDS-type1 is to find all the family of maps with isomorphic structure in which one particular system can be studied from other system, preserving the global conditions imposed.

First we must known that for topological space, it does not exist disjoin sets, under the presence of weak mixing and unbounded complexity (non equicontinuous) associate to n-cover (For all type of n-weaker mixing maps).

In a MDS-type1, there exists a measure μ Borel T-invariant, and therefore elements in non-wandering sets. For non minimal system we can see T-invariant, but not in Devaney subsystem (connection broken between ω-limits and positive h_T)

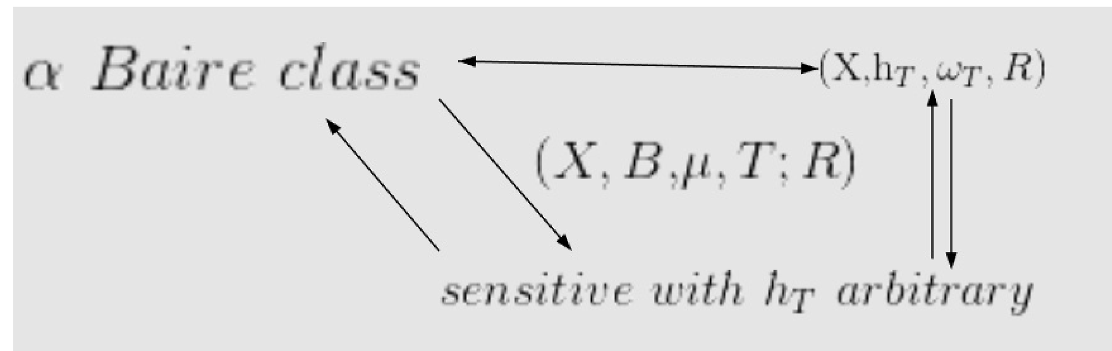
X is isomorphic to all set with horse-shoe typical maps, when there's not exists a minimal zero topological entropy, and in this cases, for some connection between a metric space with Hausdorff metric, X is Baire category 2 (trivial)

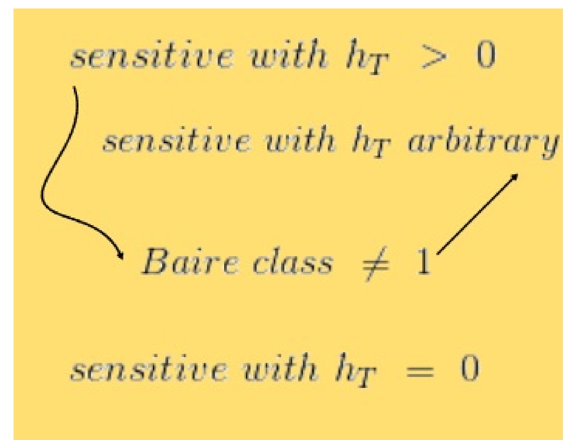
From the point of view of the existence of non-isolated, regular and clearly uncountable sets in X, we can study a dynamic associated with a family of compact subsets, in which locally taking, we can evaluate the chaotic behaviour under the assumption that exist ergodic subsystems linked to the S-system. (Here the connection between Li-York, Bruckner-Ceder and Devaney map is favorable to describe locally the behaviour of the sensitive systems using a sequence of extension maps

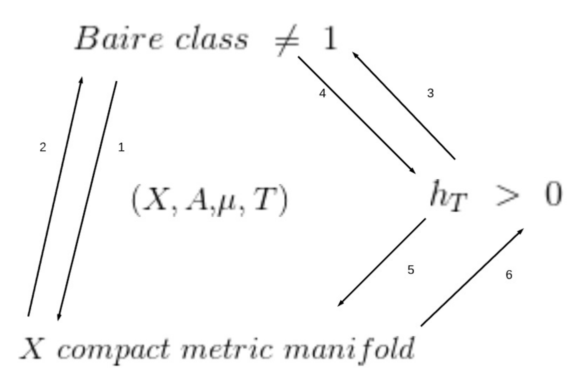
Relation Li-York system with MDS-type1

As we can known now.

- In this chapter, we going to consider some important characters about the most globally study of a system. First the case of topological entropy that imply DC2 with shadowed properties and therefore given to one analyser more posibilities to study this system using metric spaces linked to Hausdorff metric and a collection of compact subspace isomorphic to an unit interval, but don't be homeomorphic in all the cases.
- The presence of a weak mixing system in a Li-York system is a interesant condition in which we cannot use extension maps for study the evolution of sensitive system under higher dimensions, and therefore the properties of sensitive maps depends of inner condition, being all relative properties in Devaney chaotic subsystems.
- Our question in this section is about we can write a family of continuous maps as $C(X, \pi_1^{-1}Y, ...)$ for an set of homeomorphic map π_i for all i.

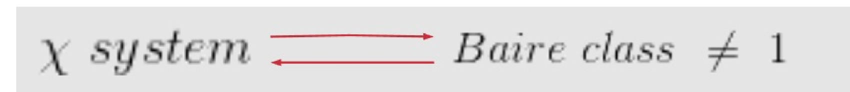


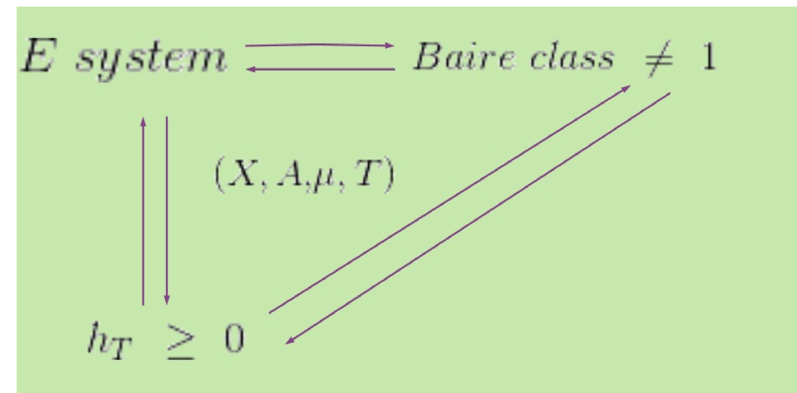




E-system, positive h_T and ω-limit

1. You can define E-systems in shift maps restricted by a perfect subset in a space sequence and being T a Devaney chaotic, with the limit set everywhere discontinuous on X. This imply that the sensitive system has not a strict relation with positive topological entropy, such that the entropy is zero when the perfect subset are T-invariant and the limit set is everywhere discontinuous.
2. If X is isomorphic to the unit interval, then the topological entropy must be zero for an infinite limit set and being T in X a Lie and York chaotic map but is not Devaney. Also you can see that a periodic set is not containing in a limit set, thus it's so doubtful about (X,T) has E-system, but that's no necessary strong to say that (X,T) has E-system, because a P-system can be infinite or disconnected if we have not Baire space on manifolds locally isomorphic to unit intervals on Devaney system.
3. Naturally we can built E-systems if there exists a MDS equivalent to a MDS-type 1
4. If a MDS-type1 has a map T, with zero topological entropy as minimum (from the sense of Devaney, clearly), then all isomorphic Baire space with a Cantor space of BCT2 must be MDS-type 1
5. If a E-system has positive topological entropy, then is a χ-system and the P-system is not infinite, then also the M-system





1. If X is transitive and perfect, therefore closed and uncountable, then the limit set has Baire class 2 and is a χ-system
2. Let (X,T) a chaotic system with ω_T having Baire class 2, then we say that (X,T) is an E-system with $h_T > 0$ if there exist a T-invariant element with $FixT^n$ dense o

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