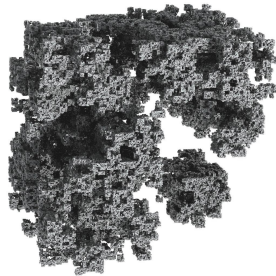


Program and Abstracts of the International Conference

Fractal Geometry and Stochastics 6

Bad Herrenalb (Black Forest)

30 Sep – 5 Oct 2018



DFG

 **KIT**
Karlsruhe Institute of Technology

SimTech 
Cluster of Excellence

WELCOME

Dear Colleagues,

Welcome to Bad Herrenalb and thank you for joining “*Fractal Geometry and Stochastics 6*”!

This conference is the sixth meeting within the conference series under this name. Since its initiation in 1994 by Christoph Bandt, Siegfried Graf and Martina Zähle the series has grown into a leading platform that connects researchers working in fractal geometry and related fields.

Unlike more specialized meetings, our conference follows the tradition of the series and aims to represent a broad spectrum of topics including

- Classical fractal geometry (dimension theory, geometric measure theory, structure of fractals)
- Analysis, stochastics and mathematical physics on fractals and metric measure spaces
- Stochastic models with fractal properties - in particular networks, graphs and trees
- Dynamical systems and ergodic theory
- Multifractals and local dimension theory
- Random geometries and random fractals.

We are proud to present 12 keynote and 15 invited talks by highly renowned experts and promising young talents who will comment on exciting new trends and latest developments in their fields and explain their own recent research. In addition there will be three parallel sessions hosting selected contributed talks on a wide range of topics, and a specially featured poster session.

Our primary goal is to connect researchers sharing similar interests, regardless of their background or career stage. Any fruitful mathematical conversation between friends newly made or reunited old friends and any new idea for joint projects will be a great success.

We are delighted to have you all on board and expect your contributions with curiosity and excitement. We are looking forward to the week and wish you a productive meeting and an enjoyable stay in the scenic Black Forest !

Uta Freiberg, Ben Hambly, Michael Hinz and Steffen Winter

GENERAL INFORMATION

Conference Venue

Our conference venue **Haus der Kirche** is a conference center of the Protestant Church of Germany. It is situated at *Bad Herrenalb*, a health resort in the northern part of the Black Forest. The town centre with its spa gardens, mineral springs and some remains of a medieval monastery as well as the train station are all within walking distance from the conference venue. The street address is *Evangelische Akademie Baden, Dobler Str. 51, 76332 Bad Herrenalb, Germany*.

For a street map of Bad Herrenalb see the inside of the front cover.

Meals, Drinks and Conference Dinner

Breakfast is served from 7.30 to 9.00am. For participants accommodated at *Hotel Sonnenhof* or *Hotel Harzer* breakfast is served in their hotel.

Lunch is served at 12.30 and **dinner** at 6.30pm.

During **coffee breaks** water, coffee and tea will be available. Water will also be available during all meals. Other drinks are available at any time but are not covered. There are several fridges and shelves with drinks throughout the house with an honesty box beside them. It is also possible to get a sheet of paper from the reception desk on which you can mark your consumed drinks and pay when you check out.

If you have any special **dietary requirements** or food allergies, please discuss these directly with the hotel staff or let us know.

On Thursday evening you are cordially invited to a **conference dinner barbecue** starting at 7pm to celebrate a hopefully inspiring and enjoyable conference.

Wireless Internet Access

Wireless internet access is provided free of charge for all participants within the conference venue. The name of the network is *HdK-Hotspot* and the network key is *HdK47110*.

Participants accommodated at *Hotel Sonnenhof* or *Hotel Harzer* enjoy also free wireless internet access in their hotels. Please request access details at the reception desk of your hotel.

Session Format and Talk Style

For the sake of lively discussion, please respect the following maximal talk times allocated to your presentations:

Keynote lectures: 45 minutes talk time;
Invited lectures: 30 minutes talk time;
Contributed talks: 15 minutes talk time.

All lecture rooms are equipped with a computer and a video projector. There is also some black or white board available in each lecture room. We recommend to prepare slides as the boards may be too small to rely on them for a whole talk. Please upload the slides of your talk (preferably in pdf-format) to the computer in the lecture room well before the start of the session allocated to your presentation. It is also possible to send the file by email to fgs6@math.kit.edu (preferably on the day before your session).

Poster Session

Posters are displayed for the duration of the conference, they are also available online at

<http://fgs6.math.kit.edu/72.php>

There is a dedicated **Poster Session** on Tuesday from 8pm to 9:30pm. Abstracts of the posters can be found in this booklet starting from page 63.

Please take note of the **Best Poster Award** sponsored by the Birkhäuser publishing house. Your conference booklet contains a ballot slip for your choice of the three best posters. Please hand in your completed ballot slip by the coffee break on Wednesday morning. The ballot box will be placed in the main lecture hall.

Monday, 1 October 2018

Chair: Bandt

8:50 – 9:00 **Welcome**

9:00 – 9:50 **KN Fraser**

Interpolating between dimensions

9:50 – 10:25 **IT Bárány**

Dimension of planar self-affine measures with application to Birkhoff and Lyapunov spectra

coffee break

11:00 – 11:35 **IT Hochman**

Dimension of self-affine measures: overlapping and non-planar cases

11:35 – 12:25 **KN Seuret**

Function spaces in multifractal environment, and the Frisch-Parisi conjecture

lunch break

Chair: K. Simon

14:00 – 14:50 **KN Miller**

The Tutte embedding of the mated-CRT map converges to Liouville quantum gravity

14:50 – 15:25 **IT Lehrbäck**

Assouad type dimensions: Examples and applications

coffee break

16:00 – 18:10 **Parallel Sessions**

(see next page)

dinner

20:00 – 20:20 **CT Mekhontsev**

Databases of self-affine sets and tiles 1. Search and analysis algorithms

20:20 – 20:40 **CT Bandt**

Databases of self-affine sets and tiles 2. Neighbor graphs and neighborhood graphs

Talks (Monday, 1 October 2018)

Parallel Session 1 (Lecture Hall)

16:00 – 16:20	Howroyd	<i>On the Hausdorff dimension of microsets</i>
16:20 – 16:40	Yu	<i>Equidistributed sequences, Bernoulli decomposition, number theory and fractal geometry</i>
16:40 – 17:00	Nikiforov	<i>Essentially non-normal numbers for Cantor series expansions</i>
short break		
17:10 – 17:30	Troscheit	<i>Self-conformal sets with positive Hausdorff measure</i>
17:30 – 17:50	Romney	<i>Inverse absolute continuity of quasiconformal and quasisymmetric mappings</i>
17:50 – 18:10	Luo	<i>On the Lipschitz equivalence of self-similar sets</i>

Parallel Session 2 (Seminar room 7)

16:00 – 16:20	Rogers	<i>Spectra of graphs related to the Basilica group</i>
16:20 – 16:40	Teplyaev	<i>Pure point spectrum on fractals and related questions</i>
16:40 – 17:00	Pietruska-Pałuba	<i>Reflected Brownian motion on nested fractals</i>
short break		
17:10 – 17:30	Ehnes	<i>Stochastic Diffusion Equations on Cantor-like Sets</i>
17:30 – 17:50	L. Simon	<i>Fractal analysis and fixed point theorems on probabilistic high-dimensional Apollonian graph sequences</i>

Parallel Session 3 (Seminar room 8)

16:00 – 16:20	Falconer	<i>Self-Stabilising processes</i>
16:20 – 16:40	Ayache	<i>Almost sure approximations in Hölder norms of a general stochastic process defined by a Young integral</i>
16:40 – 17:00	Kolossváry	<i>Triangular Gatzouras-Lalley-type planar carpets with overlaps</i>
short break		
17:10 – 17:30	Barral	<i>Dimensions of statistically self-affine Sierpinski sponges</i>
17:30 – 17:50	Soos	<i>Random fractals and measures for singlevalued and multivalued generalized contractions</i>
17:50 – 18:10	Sönmez	<i>On the carrying dimension of occupation measures coinciding with the graph dimension</i>

KN

Jonathan Fraser

09:00 – 09:50

Interpolating between dimensions**Jonathan Fraser***(University St. Andrews)*

One of the most fascinating aspects of dimension theory is how different notions of dimension relate to each other. There are several distinct dimensions available to us and they provide different information about the geometry of a given fractal set. Rather than study these notions in isolation I propose a couple of approaches to ‘interpolating between dimensions’, that is, defining a parameterised dimension function which (ideally) varies continuously between two given dimensions. The philosophy here is that the nature of the interpolation yields more nuanced information about the fractal set, the given dimensions, and the problem being considered. I will discuss several examples.

IT

Balázs Bárány

09:50 – 10:25

Dimension of planar self-affine measures with application to Birkhoff and Lyapunov spectra**Balázs Bárány***(Budapest University of Technology)*

For affine iterated function systems on the plane, under the assumption of strong irreducibility and strong open set condition, we prove that the Hausdorff dimension of the attractor is equal to the affinity dimension, and similarly for self-affine measures, the dimension is equal to the Lyapunov dimension.

Under the assumptions above, we calculate the Birkhoff spectrum of continuous potentials and the Lyapunov spectrum.

The talk is based on two papers, which are joint works with Michael Hochman, Ariel Rapaport and Thomas Jordan, Antti Käenmäki, Michał Rams.

IT

Mike Hochman

11:00 – 11:35

Dimension of self-affine measures: overlapping and non-planar cases**Mike Hochman***(The Hebrew University)*

I will discuss a recent project with Ariel Rapaport, building on our joint work with Balázs Bárány, in which we compute that dimension of self-affine sets and measures under very mild non-triviality assumptions on the matrices in the IFS. The aim of the talk is to explain why the situation in \mathbb{R}^3 , even with separation assumptions, requires an understanding of the overlapping case in \mathbb{R}^2 ; and what the challenges are in this problem. As time permits, I will discuss some of the ingredients in the proof. This talk continues the themes from Balázs Bárány's talk, but will be self-contained.

KN

Stéphane Seuret

11:35 – 12:25

Function spaces in multifractal environment, and the Frisch-Parisi conjecture

Stéphane Seuret

(University of Paris Est)

Multifractal properties of data, especially in turbulence, are now seriously established. Unfortunately, the parameters measured on these data do not correspond to any typical (or almost sure) properties of functions in the standard functional spaces: Hölder, Sobolev, Besov... In this talk, we introduce very natural function spaces in which the typical functions possess very rich scaling properties, mimicking those observed on data for instance. We obtain various characterizations of these function spaces, in terms of oscillations or wavelet coefficients. The results we prove provide us with a solution to the so-called Frisch-Parisi conjecture.

This is a joint work with Julien Barral (Université Paris 13).

KN

Jason Miller

14:00 – 14:50

The Tutte embedding of the mated-CRT map converges to Liouville quantum gravity

Jason Miller

(University of Cambridge)

Liouville quantum gravity (LQG) is a canonical model for a random surface which was introduced by Polyakov in the 1980s. It has long been conjectured that LQG describes the large scale behavior of different types of random planar maps, discrete models of random surfaces which go back to work of Tutte in the 1960s, embedded into the plane in a conformal manner. We will describe a family of random planar map models called mated-CRT maps which arise by gluing together a pair of continuum random trees (CRTs) and show that their

Tutte embedding (a.k.a. barycentric embedding) converges to Liouville quantum gravity.

Based on joint work with Ewain Gwynne and Scott Sheffield.

IT

Juha Lehrbäck

14:50 – 15:25

Assouad type dimensions: Examples and applications

Juha Lehrbäck

(University of Jyväskylä)

Let E be a set in a metric space. In the definitions of Assouad type dimensions one is asking how many balls of radius r are needed (at most or at least) to cover the sets $E \cap B(x, R)$, for any $x \in E$ and $0 < r < R < \text{diam}(E)$. However, these notions have several other characterizations and also the terminology is a bit varied; for instance, the names upper/lower Assouad dimension, uniform metric dimension and lower dimension are used in the literature.

The purpose of this talk is to present some recent results related to these dimensions and their connections to various geometric and analytic properties of sets. These topics include the Assouad dimensions of inhomogeneous self-similar sets and Muckenhoupt A_p -properties of the distance weights $\text{dist}(y, E)^{-\alpha}$, for $\alpha \in \mathbb{R}$. The talk is based on my joint work with several co-authors.

On the Hausdorff dimension of microsets

Douglas Howroyd
(University of St Andrews)

We investigate how the Hausdorff dimensions of microsets are related to dimensions of the original set. It is known that the maximal dimension of a microset is the Assouad dimension of the set. We prove that the lower dimension can analogously be obtained as the minimal dimension of a microset. In particular, the maximum and minimum exist. We also show that for an arbitrary set $\Delta \subseteq [0, d]$ containing its infimum and supremum there is a compact set in $[0, 1]^d$ for which the set of Hausdorff dimensions attained by its microsets is exactly equal to the set Δ . Our work is motivated by the general programme of determining what geometric information about a set can be determined at the level of tangents. This is joint work with Jonathan Fraser, Antti Käenmäki and Han Yu.

**Equidistributed sequences, Bernoulli decomposition,
number theory and fractal geometry**

Han Yu
(University of St Andrews)

Given an equidistributed sequence $\{x_k\}_{k \geq 1}$ in $[0, 1]$, we want to select each $x_k, k \geq 1$ independently with probability $p > 0$. We show that the randomly selected sequence is almost surely equidistributed. This simple probability model has some far-reaching applications. As an example in number theory, we shall see that the sequence $\{p(n) + 2^n d \pmod{1}\}_{n \geq 1}$ has full box dimension for an arbitrarily chosen number d and any polynomial p with at least one irrational coefficient of non-constant terms. For fractal geometry, we shall see that any non-trivial slice (not parallel with the coordinate axis) of $A_2 \times A_3$ is small in a certain sense, where A_2, A_3 are closed $\times 2 \pmod{1}$ and $\times 3 \pmod{1}$ invariant sets respectively. If we interpret 'small' in terms of having zero upper box dimension this is a result due to Shmerkin and Wu separately. We shall give some improvements by showing that those slices are small in a certain topological sense which is stronger than having zero fractal dimensions.

Essentially non-normal numbers for Cantor series expansions

Roman Nikiforov

(*Dragomanov National University*)

Denote by $N_n^b(B, x)$ the number of times a block B occurs with its starting position no greater than n in the b -ary expansion of x .

A real number x is *normal in base b* if for all k and blocks B in base b of length k , one has

$$\lim_{n \rightarrow \infty} \frac{N_n^b(B, x)}{n} = b^{-k}. \quad (1)$$

A number x is *simply normal in base b* if (1) holds for $k = 1$.

Borel introduced normal numbers in 1909 and proved that almost all (in the sense of Lebesgue measure) real numbers are normal in all bases. Obviously that the complement of the set of normal numbers has zero Lebesgue measure. But how small is the complement in fractal and topological sense?

Let consider a subset of set of non-normal numbers for which limit (1) does not exist for any individual digit. Such numbers called essentially non-normal numbers. It was proven by Albeverio, Pratsiovytyi and Torbin in 2005 that this set has full Hausdorff dimension and is of second Baire category. This result was extended for different system of numeration with finite alphabet (Q -expansion, Q^* -expansion) and with infinite alphabet (Q_∞ -expansion, I - Q_∞ -expansion, Lüroth series expansion). We extend and generalize this result for large class of Cantor series expansion considering numbers for which limit does not exist for any block of digits for all k . Furthermore the result still hold for the set of essentially non-normal numbers whose Cantor series digits are sampled along all arithmetic progressions.

This is a joint work with Dylan Airey and Bill Mance.

Self-conformal sets with positive Hausdorff measure

Sascha Troscheit

(*University of Waterloo*)

In this talk we consider sets that satisfy a slightly stronger version of quasi self-similarity that is satisfied by self-conformal sets and their graph-directed

extensions. We show that any Hausdorff measurable subset of such sets has uniformly comparable Hausdorff measure and Hausdorff content. This has many interesting consequences, especially when restricting to subsets of the real line. For those we prove that Ahlfors regularity is equivalent to the weak separation condition, which, in turn, allows us to prove a self-conformal extension of the dimension drop conjecture for self-conformal sets with positive Hausdorff measure: we show that its Hausdorff dimension falls below the expected value if and only if there are exact overlaps. (Joint work with Antti Käenmäki)

Parallel Session 1

Matthew Romney

17:30 – 17:50

Inverse absolute continuity of quasiconformal and quasisymmetric mappings

Matthew Romney
(*University of Jyväskylä*)

Quasiconformal mappings and their close relatives, quasisymmetric mappings, are important classes of geometry-preserving mapping in complex analysis, with applications to fields such as complex dynamics and geometric group theory. In this talk, we consider the problem of to what extent quasiconformal and quasisymmetric mappings can distort the dimension of large subsets. Specifically, we construct a metric space (X, d) and a quasisymmetric mapping from \mathbb{R}^n to X which maps a set of positive Hausdorff n -measure onto a set of arbitrarily small positive Hausdorff dimension. In fact, this mapping may be Lipschitz. For the case $n = 2$, we show that this construction may be realized as a quasiconformal homeomorphism of \mathbb{R}^3 . That is, the mapping takes a subset of $\mathbb{R}^2 \times \{0\}$ of positive Hausdorff 2-measure onto a set of small Hausdorff dimension. Our work answers a set of related problems posed, in various forms, by Gehring, Väisälä, Heinonen–Semmes, and Astala–Bonk–Heinonen. Portions of this work are joint with D. Ntalampekos.

Parallel Session 1

Jun Luo

17:50 – 18:10

On the Lipschitz equivalence of self-similar sets

Jun Luo
(*Chongqing University*)

Kenneth Falconer ever said: Lipschitz equivalence in fractal geometry is as important as the topological equivalence in topology. Recently there have been a lot of studies on the Lipschitz equivalence of self-similar sets under various si-

tuations. In this talk, we would like to give a short review on the progress of this topic and explain several technical methods developed.

Parallel Session 2

Luke Rogers

16:00 – 16:20

Spectra of graphs related to the Basilica group

Luke Rogers

(University of Connecticut)

We consider a sequence of graphs related to the Schreier graphs and orbital Schreier graphs of the Basilica group. The latter are related to the fractal blow-ups of Strichartz. Grigorchuk and Zuk used self-similar group methods to prove that these spectra could be understood as the intersection of the Julia set of a two-dimensional dynamical system with a constraint equation. We consider a different approach to obtain a dynamical description of the spectrum, obtaining results on the structure of the spectral measure and describing orbital Schreier graphs for which the spectrum is pure point.

The results are part of joint work with A. Brzoska, C. George, S. Jarvis and A. Teplyaev.

Parallel Session 2

Alexander Teplyaev

16:20 – 16:40

Pure point spectrum on fractals and related questions

Alexander Teplyaev

(University of Connecticut)

The talk will describe the ubiquitous generic appearance of pure point spectrum on many families of infinite fractals (unbounded infinite blow-ups) and related infinite graphs. The basic examples include the infinite Sierpinski gasket and similar nested fractals, which appeared in the works of Barlow, Kigami, Malozemov, Sabot, Strichartz et al. More recent examples include the spectral analysis on the symmetric Barlow-Evans fractals, also called vermiculated spaces, in joint works with Ben Steinhurst and with Patricia Alonso-Ruiz, Fabrice Baudoin, Dan Kelleher. Another recent example is the Basilica Julia set and the related Schreier graphs of the Basilica group, based on a joint work with Luke Rogers, Antoni Brzoska, Courtney George, Samantha Jarvis. This is connected to the recent work in group theory by Bartoldi, Grigorchuk, Nagnibeda, Nekrashevych, Sunic, Zuk et al. The talk will explain the geometric reasons for the pure point spectrum on fractals and explore analytic, physical and probabilistic implications, such as the

oscillations of the heat kernels and other spectral and probabilistic quantities.

Parallel Session 2

K. Pietruska-Pałuba

16:40 – 17:00

Reflected Brownian motion on nested fractals

Katarzyna Pietruska-Pałuba

(University of Warsaw)

For infinite nested fractals that have the so-called ‘good labeling property’, we construct the reflected Brownian motion on its compact sub-fractals and prove its properties. Such a process, previously constructed on the Sierpiński gasket, is needed for problems related to random Schrödinger operators on infinite fractals.

The talk will be based on the paper: Kamil Kaleta, Mariusz Olszewski, Katarzyna Pietruska-Pałuba, Reflected Brownian on simple nested fractals, available at <https://arxiv.org/pdf/1804.04228>

Parallel Session 2

Tim Ehnes

17:10 – 17:30

Stochastic Diffusion Equations on Cantor-like Sets

Tim Ehnes

(Universität Stuttgart)

We consider stochastic diffusion equations defined by fractal Laplacians on Cantor-like sets. After giving an estimate on the uniform norm of the eigenfunctions and resulting heat kernel estimates, we formulate conditions which ensure the existence and uniqueness of mild solutions as well as the spatial and temporal continuity. Further, we calculate the Hölder exponents and compare with the exponents in case of the classical and the p.c.f. Laplacian.

Parallel Session 2

Levente Simon

17:30 – 17:50

Fractal analysis and fixed point theorems on probabilistic high-dimensional Apollonian graph sequences

Levente Simon

*(Eötvös Loránd University Budapest, Hungary and
Babes Bolyai University, Cluj)*

In this talk, we highlight fractal analysis results on infinite high-dimensional Apollonian graphs using the Ndim algorithm introduced by Hahn, Massopust and Prigarin.

We extend the high-dimensional Apollonian network model case with a probabilistic parameter and we calculate the fractal dimension values for specific cases.

We define iterated function systems on the set of the high-dimensional Apollonian networks and we generate the growing Apollonian graph sequences based on these systems. Based on a weighted graph edit distance, we also show that infinite graphs can be also interpreted as fixed points.

- [1] K. Hahn, P. Massopust, S. Prigarin *A new method to measure complexity in binary or weighted networks and applications to functional connectivity in the human brain*, BMC Bioinformatics. (2016), ID 17:87.
- [2] Sanfeliu, A., Fu, K.-S., A distance measure between attributed relational graphs for pattern recognition, *IEEE Transactions on Systems, Man and Cybernetics*. **13**(1983), 3: 353-363.
- [3] Z. Zhang, F. Comellas, G. Fertin, L. Rong., High dimensional Apollonian networks. *Journal of Physics A: Mathematical and General*, **39**(2006), ID: 1811.

Parallel Session 3

Kenneth Falconer

16:00 – 16:20

Self-Stabilising processes

Kenneth Falconer

(University of St Andrews)

Self-Stabilising processes are stochastic processes which are locally self-similar with local form that of an α -stable process, where α depends on the value of the (left limit) of the process at the time. The talk will discuss the construction and properties of such processes. This is joint work with Jacques Lévy Véhel.

Parallel Session 3

Antoine Ayache

16:20 – 16:40

Almost sure approximations in Hölder norms of a general stochastic process defined by a Young integral

Antoine Ayache

(Lab. Painlevé, University of Lille)

We focus on a stochastic process Y defined by a pathwise Young integral of a general form. Thanks to the Haar basis, we connect the classical method of

approximation of Y through Euler scheme and Riemann-Stieltjes sums with a new approach consisting in the use of an appropriate series representation of Y . This representation is obtained through a general compactly supported orthonormal wavelet basis. An advantage offered by the new approach with respect to the classical one is that a better almost sure rate of convergence in Hölder norms can be derived, under a general chaos condition. Also, this improved rate turns out to be optimal in some situations; typically, when the integrand and integrator associated to Y are independent fractional Brownian motions with appropriate Hurst parameters.

This a joint work with Céline Esser (Université de Liège, Belgium) and Qidi Peng (Claremont Graduate University, USA)

Parallel Session 3

István Kolossváry

16:40 – 17:00

Triangular Gatzouras-Lalley-type planar carpets with overlaps

István Kolossváry

(Alfréd Rényi Institute of Mathematics)

We construct a family of planar self-affine carpets using (lower) triangular matrices in a way that generalizes the original Gatzouras–Lalley carpets defined by diagonal matrices. Of particular interest are overlapping constructions, where we allow complete columns to be shifted along the horizontal axis or allow parallelograms to overlap within a column in a transversal way. Our main result is to show sufficient conditions under which these overlaps do not affect the dimension of the attractor. Furthermore, we also consider the appropriate dimensional Hausdorff measure of the attractor. Several examples are provided to illustrate the results. Joint work with Károly Simon.

Parallel Session 3

Julien Barral

17:10 – 17:30

Dimensions of statistically self-affine Sierpinski sponges

Julien Barral

(Paris 13 University)

Gatzouras and Lalley obtained the Hausdorff and box counting dimensions of statistically self-affine Sierpinski carpets. The situation turned out to show new features with respect to the deterministic Bedford-McMullen carpets. We will explain an alternative approach which makes it possible to deal with the higher dimensional case.

This is joint work with De-Jun Feng.

Parallel Session 3

Anna Soos

17:30 – 17:50

Random fractals and measures for singlevalued and multivalued generalized contractions

Anna Soos

(Babes Bolyai University)

We will present new results in fixed point theory for random contractions for generalized singlevalued and multivalued cases. The case of coupled random fractal set will be treated. Applications for random self similar sets and measures will be discussed.

Parallel Session 3

Ercan Sönmez

17:50 – 18:10

On the carrying dimension of occupation measures coinciding with the graph dimension

Ercan Sönmez

(Heinrich Heine Universität Düsseldorf)

We investigate an interesting relation to a series of articles by U. Zähle (1988-1991) to current problems of interest concerning fractal path properties of a wide class of so-called self-affine random fields.

Databases of self-affine sets and tiles 1. Search and analysis algorithms

Dmitry Mekhontsev

(Sobolev Institute of Mathematics, Novosibirsk)

This talk explains the methods used in the software package IFStile finder, available at ifstile.com. We work with graph-directed self-similar sets $C_j = \bigcup \{f_k(C_j) \mid (j, k) \in Q_i\}$ where the f_k are maps on \mathbb{R}^n which involve the inverse of an integer matrix G , integer translations and maps from a symmetry group, like rotations and reflections. It is assumed that G is expanding on a invariant linear subspace. A main problem is to find examples with open set condition or weak separation condition, and other nice properties, for instance sets with non-empty interior, which generate tilings.

The structural equations, the matrix G and the symmetry group characterize an IFS family. They are taken as initial data. The program will find those combinations of translations and symmetry maps which generate finite type examples. Parameters are changed by a kind of random walk. A neighbor graph is constructed for every new instance, and if the number of vertices is smaller than a given number, say 100, the example is a candidate for the list of results.

An important problem is to recognize equivalent examples, so that they will be listed only once. To this end, various properties of the examples are determined: the dimensions and measures of boundary sets, properties of the neighbor graph, moments of the natural measures and combinatorial properties of the sets. The function of the package will be demonstrated. In some families, thousands of different examples can be generated within few seconds. Other families are rather small and require extensive search. Problems for future research will be discussed. This is joint work with C. Bandt.

Databases of self-affine sets and tiles 2. Neighbor graphs and neighborhood graphs

Christoph Bandt

(University of Greifswald)

The neighbor graph of a self-similar set $C = \bigcup f_k(C)$ is a tool to study the topological and geometric structure of C . For the case when all f_k have the same contraction factor, and in particular for self-similar tilings, this tool is well known under different names. It describes the graph-directed system of all boundary sets of C . We generalize the technique to the case of different contraction factors, and to graph-directed systems with several attractors C_i . As in the basic case, a neighbor graph contains the complete information on the neighbor graph of the C_i . While the calculation of neighbor graphs by hand seems difficult, efficient algorithms make it possible to deal with graphs with hundreds of vertices within milliseconds.

From the neighbor graph, the graph of neighborhoods can be derived. In the case of tilings, it is usually smaller than the neighbor graph. In the case of fractals, it can be very large. Surprising examples will be derived by small modification of the Sierpinski gasket. The main point is that the neighborhood graph is a discrete version of the magnification flow of the fractal. Self-similar sets are the linear objects of fractal geometry. In case of the finite type condition, their tangential structure becomes apparent from finite-size magnification. Nevertheless, it can be unexpectedly complicated. In this setting, certain ergodic averages can be determined by simple matrix calculation. This is joint work with D. Mekhontsev.

Tuesday, 2 October 2018

Chair: Falconer

9:00 – 9:50 **KN Peres**

Keakeya sets from a search game and from Brownian motion

9:50 – 10:25 **IT Sava-Huss**

Cluster growth models on the Sierpinski gasket

coffee break

11:00 – 11:50 **KN Akkermans**

Quantum symmetry breaking : Scale anomaly and fractals

11:50 – 12:25 **IT Hilfer**

Multiscale Local Porosity Theory, Weak Limits and Dielectric Response in Composite and Porous Media

lunch break

Chair: M. Järvenpää

14:00 – 14:50 **KN Shanmugalingam**

Notions of functions of bounded variation on metric spaces: using upper gradients versus Dirichlet forms

14:50 – 15:25 **IT Smirnova-Nagnibeda**

Spectra of graphs associated with self-similar groups

coffee break

16:00 – 18:10 **Parallel Sessions**

(see next page)

dinner

20:00 – 21:30 **Poster Session**

Parallel Session 1 (Lecture Hall)

16:00 – 16:20	Allen	<i>A general mass transference principle</i>
16:20 – 16:40	Baker	<i>Maximising Bernoulli measures and dimension gaps for countable branched systems</i>
16:40 – 17:00	K. Simon	<i>Hausdorff dimension for some non-Markovian repellers which were inspired by fractal image compression</i>

short break

17:10 – 17:30	Tetenov	<i>Topologically self-similar dendrites, generated by m-sprouts</i>
17:30 – 17:50	Frettlöh	<i>Fractal bounded remainder sets</i>
17:50 – 18:10	Yavicoli	<i>Fractals and patterns</i>

Parallel Session 2 (Seminar room 7)

16:00 – 16:20	Chen	<i>The abelian sandpile growth problem on the Sierpinski gasket is exactly solved</i>
16:20 – 16:40	Hattori	<i>Displacement exponents for loop-erased random walks on the Sierpinski gasket and the 3-gasket</i>
16:40 – 17:00	Rousselin	<i>Dimension drop for random walks on random trees</i>

short break

17:10 – 17:30	Steinhurst	<i>Spectral Segmentation in Nearly-Self-Similar Laakso spaces</i>
17:30 – 17:50	Landry	<i>Spectral Triples, Quantum Compact Metric Spaces, and the Sierpinski Gasket</i>
17:50 – 18:10	Post	<i>On an abstract convergence scheme for discrete energy forms approximating energy forms on metric spaces</i>

Parallel Session 3 (Seminar room 8)

16:00 – 16:20	Järvenpää	<i>Hausdorff dimension of limsup sets of rectangles in the Heisenberg group</i>
16:20 – 16:40	Troshin	<i>Sierpiński gasket via IFS on Lobachevskii plane</i>
16:40 – 17:00	Rams	<i>Badly approximable numbers for irrational rotation</i>

short break

17:10 – 17:30	Buczolich	<i>Isentropes and Lyapunov exponents</i>
17:30 – 17:50	Pearse	<i>Continuity of entropy of certain Lorenz maps</i>
17:50 – 18:10	Wang	<i>Dynamics of Newton maps</i>

KN

Yuval Peres

09:00 – 09:50

Keakeya sets from a search game and from Brownian motion

Yuval Peres*(Microsoft)*

A planar set that contains a unit segment in every direction is called a Keakeya set. These sets have been studied intensively in geometric measure theory and harmonic analysis since the work of Besicovich (1919); we find a connection to game theory and probability. A hunter and a rabbit move on an n -vertex cycle without seeing each other until they meet. At each step, the hunter moves to a neighboring vertex or stays in place, while the rabbit is free to jump to any node. Thus they are engaged in a zero sum game, where the payoff is the capture time. We show that every rabbit strategy yields a Keakeya set; the optimal rabbit strategy is based on a discretized Cauchy walk, and it yields a Keakeya set consisting of $4n$ triangles, of minimal area among such Keakeya sets. Passing to the scaling limit yields a simple construction of a random Keakeya set with zero area from two Brownian motions (joint work with Y. Babichenko, R. Peretz, P. Sousi and P. Winkler). In the second (unrelated) part of the talk, I will describe an open problem: Given two independent Brownian motions, find the largest dimension of a set in $[0, 1]$ on which both of them are nondecreasing. For one Brownian motion the answer is $1/2$ (Balka-P.), but for two we only know it is between $1/3$ and $1/2$ (Balka-Angel-Mathe-P.).

IT

Ecaterina Sava-Huss

09:50 – 10:25

Cluster growth models on the Sierpinski gasket

Ecaterina Sava-Huss*(TU Graz)*

We introduce two cluster growth models: a random one called internal DLA and a deterministic one called divisible sandpile model. We show that on the infinite graphical Sierpinski gasket (SG), when particles are launched from the corner vertex o of SG, the cluster in both models fills balls (centered at o in the graph metric). The results are based on a joint work with Joe Chen, Wilfried Huss, and Alexander Teplyaev.

KN

Erik Akkermans

11:00 – 11:50

Quantum symmetry breaking : Scale anomaly and fractals

Erik Akkermans

(Technion Haifa)

Scale invariance is a common property of our everyday environment. Its breaking gives rise to less common but beautiful fractal structures. At the quantum level, breaking of continuous scale invariance is a remarkable example of quantum phase transition also known as scale anomaly. The general features of this transition and the resulting fractal spectra will be presented at an elementary quantum mechanics level.

IT

Rudolf Hilfer

11:50 – 12:25

Multiscale Local Porosity Theory, Weak Limits and Dielectric Response in Composite and Porous Media

Rudolf Hilfer

(University of Stuttgart)

A mathematical scaling approach to macroscopic heterogeneity of composite and porous media is introduced. It is based on weak limits of uniformly bounded measurable functions. The limiting local porosity distributions, that were introduced in *Advances in Chemical Physics*, vol XCII, p. 299-424 (1996), are found to be related to Young measures of a weakly convergent sequence of local volume fractions. These parametrized measures are useful to compute frequency dependent complex dielectric function for multiscale media using selfconsistent local porosity approximations. The approach separates scales by scale factor functions of regular variation. It renders upscaled results independent of the shapes of averaging windows in the scaling limit.

KN

N. Shanmugalingam

14:00 – 14:50

Notions of functions of bounded variation on metric spaces: using upper gradients versus Dirichlet forms

Nageswari Shanmugalingam

(University of Cincinnati)

Analysis on fractals and non-smooth metric spaces is a field of current active research. In 2001 Michele Miranda gave a notion of functions of bounded variation

in the non-smooth setting using Lipschitz functions. In this talk we will give an overview of two further approaches, one using the notion of upper gradients, and the other using Dirichlet forms.

IT

T. Smirnova-Nagnibeda

14:50 – 15:25

Spectra of graphs associated with self-similar groups

Tatiana Smirnova-Nagnibeda

(St Petersburg University, Russia and University of Geneve)

We will discuss Laplacian spectra of graphs associated with self-similar (fractal) groups and their actions.

A general mass transference principle

Demi Allen

(University of Manchester)

The Mass Transference Principle, proved by Beresnevich and Velani in 2006, is a remarkable result which allows one to transfer Lebesgue measure statements to Hausdorff measure statements for limsup sets determined by sequences of balls in \mathbb{R}^k . This result has also been extended to deal with limsup sets determined by sequences of neighbourhoods of “approximating” planes in \mathbb{R}^k . Such results are somewhat surprising, since Hausdorff measure refines Lebesgue measure, and have a number of consequences (especially in Diophantine Approximation).

In this talk, we present a general version of the Mass Transference Principle for limsup sets determined by neighbourhoods of sets satisfying a certain “local scaling property”. Further to those results previously established, our result holds when the underlying sets are smooth compact manifolds or self-similar sets satisfying the open set condition. The generality of the statement we propose opens up a new range of possible applications to explore. This talk is based on joint work with Simon Baker (Warwick, UK).

Maximising Bernoulli measures and dimension gaps for countable branched systems

Simon Baker

(University of Warwick)

Kifer, Peres, and Weiss proved that there exists $c_0 > 0$, such that $\dim \mu \leq 1 - c_0$ for any probability measure μ which makes the digits of the continued fraction expansion i.i.d. random variables. In this talk I will discuss a recent paper with Natalia Jurga where we prove that amongst this class of measures, there exists one whose dimension is maximal. Our results also apply in the more general setting of countable branched systems.

Hausdorff dimension for some non-Markovian repellers which were inspired by fractal image compression

Karoly Simon

(Budapest University of Technology and Economics)

Joint work with Balazs Barany (Budapest) and Michal Rams (IMPAN).

The motivation of our research was to answer a fractal image compression related question asked by Michael Barnsley.

Namely, in a one dimensional image, the brightness, point by point, is represented by a function $f : I \rightarrow \mathbb{R}$, where $I := [0, 1]$. We approximate f by a function \tilde{f} whose graph is the repeller of a certain $F : I \times \mathbb{R} \rightarrow I \times \mathbb{R}$, where F is defined as follows:

Given an $N \in \mathbb{N}$ and a partition $\{I_i\}_{i=1}^N$ of I . Moreover, for every I_i we are given $F_i : I_i \times \mathbb{R} \rightarrow I \times \mathbb{R}$

$$F_i : \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} a_i & 0 \\ c_i & d_i \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} b_i \\ e_i \end{pmatrix},$$

where $a_i, b_i, c_i, d_i, e_i \in \mathbb{R}$ and $|a_i| > 1$, $|d_i| > 1$. We define

$$F(x, y) := F_i(x, y) \text{ if } x \in I_i.$$

Consider

$$R := \{(x, y) \in I \times \mathbb{R} : \{F^n(x, y)\}_{n=1}^\infty \text{ is bounded}\}.$$

Then R is the graph of a function $\tilde{f} : I \rightarrow \mathbb{R}$. Recall that this is the function with which we approximate f . The difficulty comes from the fact that (as opposed to self-affine attractors) R is not Markovian. The problem asked by Michael Barnsley was to find

$$\dim_{\text{H}}(\text{graph}(\tilde{f})) = ?$$

To solve this problem we combine techniques from one-dimensional dynamics and fractal geometry. As an application of our results we can compute the dimension of the graphs of some fractal interpolation functions and generalized Takagi functions.

Parallel Session 1

Andrei Tetenov

17:10 – 17:30

Topologically self-similar dendrites, generated by m-sprouts

Andrei Tetenov

(Gorno-Altaiisk State University/ Novosibirsk State University)

By *m-sprout* we mean a bipartite tree $\Gamma(B, W, E, \nu, \varphi)$, furnished with (injective) index function $\nu/ : \{1, \dots, m\} \rightarrow B$ and edge coloring function $\varphi : E \rightarrow \{1, \dots, m\}$. We define a composition operation $\Gamma_1 * \Gamma_2$ of m-sprouts providing a semigroup structure on the set $Sp(m)$ of all m-sprouts.

The powers Γ^n of m-sprout Γ give rise to the sequence of topological spaces X_n , associated with Γ^n . If the sprout Γ is regular, the inverse limit $X(\Gamma) = \varprojlim X_n$ is a Hausdorff space and therefore it is a topologically self-similar postcritically finite dendrite.

We prove that each self-similar p.c.f. dendrite K is homeomorphic to some $X(\Gamma)$ generated by certain m-sprout Γ where m is the cardinality of respective post-critical set. We also prove the conditions for the boundedness of ramification order of $X(\Gamma)$ and the existence of a self-similar metrics on $X(\Gamma)$.

Parallel Session 1

Dirk Frettlöh

17:30 – 17:50

Fractal bounded remainder sets

Dirk Frettlöh

(University of Bielefeld)

A (compact) subset P of $[0, 1]^d$ is a *bounded remainder set* wrt $a = (a_1, \dots, a_d)$ if $|\sum_{k=1}^n 1_P(ka \bmod 1) - n \text{vol}(P)|$ is uniformly bounded. This talk presents a connection between aperiodic point sets (cut-and-project sets) and bounded remainder sets and uses this to prove that several sets with fractal boundary are bounded remainder sets. In fact these sets with fractal boundary are the windows of the cut-and-project sets considered.

Parallel Session 1

Alexia Yavicoli

17:50 – 18:10

Fractals and patterns

Alexia Yavicoli

(University of Buenos Aires and CONICET)

I will talk about the relationship between the size of a set and the presence of geometric patterns, such as arithmetic progressions. In particular, I will show the existence of large sets that avoid countably many given linear patterns, and the existence of small sets containing a lot of geometrical configurations.

Parallel Session 2

Joe Chen

16:00 – 16:20

The abelian sandpile growth problem on the Sierpinski gasket is exactly solved

Joe Chen

(Colgate University)

Lay m chips at the corner vertex of the Sierpinski gasket graph (SG), and topple and stabilize according to the rules of the abelian sandpile model. We show that for all m , the stable cluster is always a ball in the graph metric, and the radius of the growing cluster (as a function of m) can be characterized exactly via a family of recursions. The radial function satisfies the conditions of the renewal theorem in the arithmetic case, hence asymptotically follows a power law modulated by log-periodic oscillations.

The proofs are combinatorial and rely primarily on the exact analysis of self-similar sandpile “tiles”, i.e., certain elements of the sandpile group, on subgraphs of SG. The cut point structure and the axial symmetry of SG are the main geometric inputs. In the course of our proofs we also establish the identity elements of the sandpile groups with two different boundary conditions. Notably, there is a special “Peano curve” sitting inside the identity element which plays a key role in the final part of the proofs.

This talk is a follow-up to Ecaterina Sava-Huss’ talk, and together the works we will describe establish a “limit shape universality” of Laplacian growth models on the Sierpinski gasket, in the sense of Levine and Peres (2017).

Based on joint work with Jonah Kudler-Flam, arXiv:1807.08748.

Parallel Session 2

Kumiko Hattori

16:20 – 16:40

Displacement exponents for loop-erased random walks on the Sierpinski gasket and the 3-gasket

Kumiko Hattori

(Tokyo Metropolitan University)

We prove the loop-erased random walks on two kinds of finite pre-fractals can be extended to loop-erased random walks on the infinite pre-fractals. The graphs we

consider here are the pre-Sierpinski gasket and the pre-3-gasket, and we employ the erasing-larger-loops first method for construction. We obtain the exponents governing their asymptotic behavior and prove laws of the iterated logarithm. (Partly joint work with R. Ito)

Parallel Session 2

Pierre Rousselin

16:40 – 17:00

Dimension drop for random walks on random trees

Pierre Rousselin

(LAGA, Université Paris 13)

We study different models of random walks on random trees. When the walk is transient, almost all trajectories go to infinity and define a random ray. The law of this random ray is called the harmonic measure on the boundary of this random tree. A dimension drop phenomenon occurs: this harmonic measure is almost surely carried by a small (in the Hausdorff dimension sense) part of the boundary of the tree. This theory was initiated by Lyons, Pemantle and Peres in the 90's. More recently, Curien, Le Gall and Lin have studied this phenomenon on a different model of random trees. We will see how one can generalize these results to the cases of Galton-Watson trees with recursive random lengths and Galton-Watson trees with random weights.

Parallel Session 2

Benjamin Steinhurst

17:10 – 17:30

Spectral Segmentation in Nearly-Self-Similar Laakso spaces

Benjamin Steinhurst

(McDaniel College)

Laakso spaces are convenient models to study spectral problems because for a natural Laplacian exact formulae for the (discrete) spectrum and multiplicities are available. They are available even for non-self-similar Laakso spaces. We know that this Laplacian has a continuous heat kernel and thus a well defined trace $Z(t)$ along the diagonal. For many self-similar fractals with reasonable Laplacians $Z(t)$ is known to be log-periodic near $t = 0$, this is the dual fact to the eigenvalue counting function being log-periodic as $n \rightarrow \infty$. We numerically study how the log-periodicity is effected by small perturbations to the self-similar structure. In those cases where exact computations are available those are given as well.

Parallel Session 2

Therese Landry

17:30 – 17:50

Spectral Triples, Quantum Compact Metric Spaces, and the Sierpinski Gasket**Therese Landry***(University of California, Riverside)*

One of the fundamental tools of noncommutative geometry is Connes' spectral triple. Michel Lapidus and his collaborators have developed spectral triples for the Sierpinski gasket that recover the Hausdorff dimension, the geodesic metric, and the $\log_2 3$ -dimensional Hausdorff measure. The space of continuous, complex-valued functions on the Sierpinski gasket can be viewed as a quantum compact metric space. The Gromov-Hausdorff distance is an important tool of Riemannian geometry, and building on the earlier work of Rieffel, Latrémolière introduced a generalization of the Gromov-Hausdorff distance to the quantum compact metric space. Aspects of geometry that can be recovered via the Gromov-Hausdorff propinquity in the setting of the Sierpinski gasket will be discussed and compared with the geometric information that can be obtained from spectral triples.

Parallel Session 2

Olaf Post

17:50 – 18:10

On an abstract convergence scheme for discrete energy forms approximating energy forms on metric spaces**Olaf Post***(Universität Trier, Fachbereich IV – Mathematik)*

We present an abstract scheme generalising norm resolvent convergence to the case of varying Hilbert spaces. We apply this scheme to discrete graphs with energy forms approximating energy forms on spaces such as pcf fractals, finitely ramified spaces, metric graphs or manifolds.

Parallel Session 3

Esa Järvenpää

16:00 – 16:20

Hausdorff dimension of limsup sets of rectangles in the Heisenberg group

Esa Järvenpää
(*University of Oulu*)

The almost sure value of the Hausdorff dimension of limsup sets generated by randomly distributed rectangles in the Heisenberg group is computed in terms of directed singular value functions.

Joint work with Fredrik Ekström and Maarit Järvenpää.

Parallel Session 3

Pavel Troshin

16:20 – 16:40

Sierpiński gasket via IFS on Lobachevskii plane

Pavel Troshin
(*Kazan Federal University*)

Our objective is to propose a way of constructing fractals in Lobachevskii space by means of iterated function systems (IFS) whose transformations are intrinsic to this non-Euclidean geometry.

We extend the group of motions of Lobachevskii plane to include contractions similar to homotheties on Euclidean plane, and use this extended group to construct the desired IFS. For our purposes, we use Beltrami–Klein and Poincaré models.

In this way, two versions of parametrized IFS families for Sierpiński gasket are considered together with associated Mandelbrot sets. Unlike the Euclidean case, these examples show much more complicated behavior. This happens mostly due to the transcendental nature of the introduced contraction mapping, which also results in difficulties in calculations and computing the iterations. We also face a problem when two different IFS with the same attractor on Euclidean plane generate two analogous IFS on Lobachevskii plane with different attractors.

The reported study was funded by RFBR according to the research project No 18-31-00295.

Parallel Session 3

Michał Rams

16:40 – 17:00

Badly approximable numbers for irrational rotation

Michał Rams
(*IMPAN*)

Given $\alpha, x \in [0, 1]$ let $L(\alpha, x) = \liminf_{|n| \rightarrow \infty} |n| \cdot \|n\alpha - x\|$. As proved by Kim, for every irrational number α the set $\{x : L(x, \alpha) = 0\}$ has full Lebesgue measure in $[0, 1]$. By the result of Bugeaud, Harrap, Kristensen, and Velani the complementary set $\{x : L(x, \alpha) > 0\}$ has Hausdorff dimension 1 (again, for every irrational α).

The paper by Lim, de Saxce, and Shapira was the first to investigate the set $B(\alpha, c) = \{x : L(x, \alpha) > c\}$ for varying $c \leq 0$, they proved that for almost every α $\dim_H B(\alpha, c) < 1$ for all positive c and they also gave a sufficient condition for α under which $\dim_H B(\alpha, c) = 1$ for sufficiently small c . We prove that $\dim_H B(\alpha, c) < 1$ for all $c > 0$ if and only if $\liminf_{k \rightarrow \infty} q_k^{1/k} < \infty$ (where q_k are the denominators in the continuous fraction form of α). Moreover, we prove that if $\dim_H B(\alpha, c) = 1$ for some $c > 0$ then also $\dim_H B(\alpha, 1/432) = 1$ (that is, we have a dichotomy: either $\dim_H B(\alpha, c) < 1$ for all $c > 0$ or $\dim_H B(\alpha, c) = 1$ for all $c \leq 1/432$). The number $1/432$ is not optimal, for example if $\lim_k a_k = \infty$ (where a_k are the continuous fraction coefficients of α) then $\dim_H B(\alpha, c) = 1$ for all $c \leq 1/16$.

It is a joint work with Yann Bugeaud, Dong Han Kim, and Seonhee Lim.

Parallel Session 3

Zoltán Buczolich

17:10 – 17:30

Isentropes and Lyapunov exponents

Zoltán Buczolich

(*ELTE Eötvös Loránd University*)

We consider skew tent maps $T_{\alpha, \beta}(x)$ such that $(\alpha, \beta) \in [0, 1]^2$ is the turning point of $T_{\alpha, \beta}$, that is, $T_{\alpha, \beta} = \frac{\beta}{\alpha}x$ for $0 \leq x \leq \alpha$ and $T_{\alpha, \beta}(x) = \frac{\beta}{1-\alpha}(1-x)$ for $\alpha < x \leq 1$. We denote by $\underline{M} = K(\alpha, \beta)$ the kneading sequence of $T_{\alpha, \beta}$, by $h(\alpha, \beta)$ its topological entropy and $\Lambda = \Lambda_{\alpha, \beta}$ denotes its Lyapunov exponent. For a given kneading sequence \underline{M} we consider isentropes (or equi-topological entropy, or equi-kneading curves), $(\alpha, \Psi_{\underline{M}}(\alpha))$ such that $K(\alpha, \Psi_{\underline{M}}(\alpha)) = \underline{M}$. On these curves the topological entropy $h(\alpha, \Psi_{\underline{M}}(\alpha))$ is constant.

We show that $\Psi'_{\underline{M}}(\alpha)$ exists and the Lyapunov exponent $\Lambda_{\alpha, \beta}$ can be expressed by using the slope of the tangent to the isentrope. Since this latter can be computed by considering partial derivatives of an auxiliary function $\Theta_{\underline{M}}$, a series depending on the kneading sequence which converges at an exponential rate, this provides an efficient new method of finding the value of the Lyapunov exponents of these maps.

This is a joint work with G. Keszthelyi.

Parallel Session 3

Erin Pearce

17:30 – 17:50

Continuity of entropy of certain Lorenz maps**Erin Pearce***(California Polytechnic State University)*

We consider a one parameter family of Lorenz maps indexed by their point of discontinuity p and constructed from a pair of bilipschitz functions. We prove that their topological entropies vary continuously as a function of p .

Parallel Session 3

Xiaoguang Wang

17:50 – 18:10

Dynamics of Newton maps**Xiaoguang Wang***(Zhejiang University)*

Newton's method is probably the oldest and most famous iterative process to be found in mathematics. Let p be any polynomial with at least three distinct roots, and f be its Newton map:

$$f(z) = z - \frac{p(z)}{p'(z)}.$$

It is shown that the boundary ∂B of any immediate root basin B of f is locally connected. Moreover, ∂B is a Jordan curve if and only if $\deg(f|_B) = 2$. This implies that the boundaries of all components of root basins, for all polynomials' Newton maps, from the viewpoint of topology, are tame. This generalizes Roesch's groundbreaking work (Annals of Math. 2008) on cubic Newton maps to arbitrary degree.

This work is joint with Yongcheng Yin and Jinsong Zeng.

Wednesday, 3 October 2018

Chair: Pollicott

9:00 – 9:50	KN van der Hofstad	<i>Fractal geometry critical percolation on power-law random graphs</i>
9:50 – 10:25	IT Heydenreich	<i>The Random Connection Model at Criticality</i>

coffee break

11:00 – 11:50	KN Hino	<i>Asymptotics of integrated Betti numbers for random simplicial complex processes</i>
11:50 – 12:25	IT Alonso-Ruiz	<i>Heat kernels and functional inequalities on generalized diamond fractals</i>

lunch break

13:30 – 18:30 **Excursions/ Free time**

dinner

Fractal geometry critical percolation on power-law random graphs

Remco van der Hofstad

(*TU Eindhoven*)

Empirical findings have shown that many real-world networks are scale-free in the sense that there is a high variability in the number of connections of the elements of the networks. Spurred by these empirical findings, models have been proposed for such networks. In this talk, we describe a particular class of random graphs in which edges are present independently but with unequal edge occupation probabilities that are moderated by appropriate vertex weights. For such models, it is known precisely when there is a giant component, meaning that a positive proportion of the vertices is connected to one another. We discuss the scaling limit of the metric structure of the largest connected components at criticality.

We study these problems on two popular models of complex networks, the configuration model and rank-1 inhomogeneous random graphs. Our results show that, the critical behavior admits a transition when the third moment of the degrees turns from finite to infinite. When the third moment is finite, Bhamidi, Broutin, Sen and Wang show that the largest clusters in a graph of n vertices have size $n^{2/3}$ and the metric scaling limit equals that on the homogeneous Erdős-Rényi random graph apart from trivial rescalings. In particular, the limiting metric space has Minkowski and Hausdorff dimension 2.

When the third moment of the degrees is infinite and has tails that are regularly varying with exponent $-(\tau - 1)$ with $\tau \in (3, 4)$, the largest clusters have size $n^{(\tau-2)/(\tau-1)}$, where $(\tau - 2)/(\tau - 1) \in (1/2, 2/3)$. Further, the metric scaling limit is rather different. For example, the Minkowski dimension equals $(\tau - 2)/(\tau - 3) > 2$, which can be arbitrarily large due to the presence of “hub” vertices with infinite degree. We also discuss the highly interesting and relevant setting of $\tau \in (2, 3)$, where the scaling behavior even depends on whether the model has a single-edge constraint or not.

[This is joint work with Shankar Bhamidi, Souvik Dhara, Johan van Leeuwen and Sanchayan Sen.]

IT

Markus Heydenreich

09:50 – 10:25

The Random Connection Model at Criticality

Markus Heydenreich

(LMU Munich)

The random connection model is a random graph whose vertices are given by the points of a Poisson process and whose edges are obtained by randomly connecting pairs of Poisson points in a position dependent but independent way. Under very general conditions, the resulting random graph undergoes a percolation phase transition if the Poisson density varies, and we are interested in the case of critical percolation. Our main result is an infrared bound for the critical connectivity function if the dimension is sufficiently large or if the pair connection function has sufficiently slow decay. This is achieved by devising the lace expansion for the random connection model.

Based on joint work with R. van der Hofstad, G. Last and K. Matzke.

KN

Masanori Hino

11:00 – 11:50

Asymptotics of integrated Betti numbers for random simplicial complex processes

Masanori Hino

(Kyoto University)

In this talk, we consider random processes of increasing simplicial complexes and their homological properties. The time integral of their average Betti numbers is our main interest, which is identified with the expectation of the lifetime sums of generators in terms of their persistent homologies. We determine its growth exponent as the number of vertices approaches infinity for a class of such processes, and obtain the theorem of LLN type for the Linial-Meshulam process. This result is regarded as a higher-dimensional analogue of Frieze's $\zeta(3)$ -limit theorem for the Erdős-Rényi graph process, and improves a part of the previous study by Y. Hiraoka and T. Shirai (2017). This talk is based on a joint work with Shu Kanazawa (Tohoku Univ.).

Heat kernels and functional inequalities on generalized diamond fractals

Patricia Alonso-Ruiz
(*University of Connecticut*)

Generalized diamond fractals constitute a parametric family of spaces that arise as scaling limits of so-called diamond hierarchical lattices. The latter appear in the physics literature in the study of random polymers, Ising and Potts models among others.

In the case of constant parameters, diamond fractals are self-similar sets. This property was exploited in earlier investigations by Hambly and Kumagai to study the corresponding diffusion process and its heat kernel. These questions are of interest in this setting in particular because the usual assumption of volume doubling is not satisfied.

For general parameters, also the self-similarity is lost. Still, a diamond fractal can be regarded as an inverse limit of metric measure graphs and a canonical diffusion process obtained through a general procedure proposed by Barlow and Evans. This approach will allow us to provide a rather explicit expression of the associated heat kernel and deduce several of its properties. As an application, we will discuss some functional inequalities of interest in the study of the interconnections between analytic and geometric aspects of diffusion processes.

Thursday, 4 October 2018

Chair: Zähle

9:00 – 9:50 **KN Croydon**

Scaling limits of stochastic processes associated with resistance forms

9:50 – 10:25 **IT Suomala**

Strong Marstrand's projection theorems for random sets in Euclidean and non-Euclidean spaces

coffee break

11:00 – 11:50 **KN Kern**

Semi-fractional diffusions

11:50 – 12:25 **IT Kombrink**

Renewal Theory in Fractal Geometry

lunch break

Chair: Buczolich

14:00 – 14:50 **KN Stallard**

Fractals in complex dynamics: dimensions of Julia sets and escaping sets

14:50 – 15:25 **IT Máthé**

Equidecompositions and fractal boundaries

coffee break

16:00 – 18:10 **Parallel Sessions**

(see next page)

19:00 conference dinner barbecue

Talks (Thursday, 4 October 2018)

Parallel Session 1 (Lecture Hall)

16:00 – 16:20	Rossi	<i>On measures that improve L^q dimension under convolution</i>
16:20 – 16:40	Héra	<i>Fubini-type results for Hausdorff dimension</i>
16:40 – 17:00	Farkas	<i>Interval projections of self-similar sets</i>
short break		
17:10 – 17:30	Cristea	<i>On families of labyrinth fractals</i>
17:30 – 17:50	Verma	<i>A Revisit to Fractal Interpolation Function and a Fractalization of Rational Trigonometric Functions</i>

Parallel Session 2 (Seminar room 7)

16:00 – 16:20	Kong	<i>Random walks and induced energy forms on compact doubling spaces</i>
16:20 – 16:40	Kigami	<i>Weighted partition of a compact metric space, its hyperbolicity and Ahlfors regular conformal dimension</i>
16:40 – 17:00	Ruan	<i>Metrics on the Sierpinski carpet by weight functions</i>
short break		
17:10 – 17:30	Yang	<i>Local and Non-Local Dirichlet Forms on the Sierpinski Carpet</i>
17:30 – 17:50	Hauser	<i>Oscillations on the Stretched Sierpinski Gasket</i>
17:50 – 18:10	Minorics	<i>Some Limit Theorems for the Laplacian on Statically Self-Similar Cantor Strings</i>

Parallel Session 3 (Seminar room 8)

16:00 – 16:20	Käenmäki	<i>Domination and thermodynamic formalism for planar matrix cocycles</i>
16:20 – 16:40	B. Li	<i>Metric recurrence and shrinking target theory in dynamical systems</i>
16:40 – 17:00	Telcs	<i>Inference of causal relations via dimensions</i>

KN

David Croydon

09:00 – 09:50

Scaling limits of stochastic processes associated with resistance forms

David Croydon
(*Kyoto University*)

One of the early motivations for the study of stochastic processes on fractals came from physics, where there was an interest in understanding the dynamical properties of disordered media. Specifically, certain examples of the latter were modelled by critical percolation, which is believed to exhibit large scale fractal structure. The initial response from the mathematics community was to construct Brownian motion on idealised fractals, such as the Sierpinski gasket. Since then, the technology has developed to the point where it can engage with some of the original questions about critical percolation. In this talk, I will describe recent work in this direction that underlines how the notion of a resistance form, as introduced by Kigami to analyse stochastic processes on fractals, is useful for understanding the scaling limits of various models of random walks on random graphs in critical regimes. I will also discuss applications to time-changes of random walks on fractal graphs.

IT

Ville Suomala

09:50 – 10:25

Strong Marstrand's projection theorems for random sets in Euclidean and non-Euclidean spaces

Ville Suomala
(*University of Oulu*)

In the first part of the talk, I will survey some results for spatially independent martingales in Euclidean spaces. The main focus is on slicing and projection theorems with geometric applications.

In the second part, I will discuss analogous results in two (related) non-Euclidean spaces: The Heisenberg group, and S^3 endowed with the Gromov metric.

KN

Peter Kern

11:00 – 11:50

Semi-fractional diffusions

Peter Kern
(*Heinrich Heine Universität Düsseldorf*)

The object of study are stochastic processes whose finite-dimensional marginal distributions fulfill a space-time invariance property on a discrete scale frequently called semi-selfsimilarity or self-affinity in different contexts. In the special case of processes with stationary independent increments we deal with so-called semistable Lévy processes for which in recent years some fractal path properties have been investigated explicitly in analogy to their classical stable counterparts. The main difference between a stable and a semistable Lévy process is that the power law tails of a stable Lévy measure are additionally equipped with log-periodic disturbances to get a semistable Lévy measure. This fact is used to define semi-fractional derivatives such that the probability densities of a semistable Lévy process solve a certain semi-fractional diffusion equation. We show that these semi-fractional derivative operators can be approximated by a Grünwald-Letnikov type formula which enables to solve corresponding abstract Cauchy problems numerically.

IT

Sabrina Kombrink

11:50 – 12:25

Renewal Theory in Fractal Geometry

Sabrina Kombrink

(University of Lübeck and University of Göttingen)

Classically, renewal theorems provide the leading asymptotic term of renewal functions. They have been widely applied to determine the long term behaviour of stochastic systems. Our focus lies in refining such renewal theorems by determining not only the leading asymptotic term but also asymptotic terms of lower order. Applications in Fractal Geometry include an analogue of Steiner's formula for fractal sets (concerning an asymptotic expansion of the ϵ -parallel volume as $\epsilon \rightarrow 0$) and the Weyl-Berry conjecture (concerning the eigenvalues of the Dirichlet Laplacian on domains with a fractal boundary).

KN

Gwyneth Stallard

14:00 – 14:50

Fractals in complex dynamics: dimensions of Julia sets and escaping sets

Gwyneth Stallard

(The Open University)

Complex dynamics concerns the iteration of analytic functions of the complex plane. The main objects of study include the Julia set (where the iterates have chaotic behaviour) and the escaping set (where the iterates escape to infinity). For most functions, these sets are interesting fractals and many results have

been proved on the possible values of their dimensions. This talk will give an overview of the main results and techniques in this area and discuss interesting open questions.

IT

András Máthé

14:50 – 15:25

Equidecompositions and fractal boundaries

András Máthé

(University of Warwick)

Let A and B be bounded (Borel) sets in \mathbb{R}^d having equal positive Lebesgue measure such that their boundaries have Minkowski dimension less than d . Then one can divide A into finitely many pieces and translate these so that their union gives B . (Standard example is the disc and square in the plane.) This is Laczkovich's theorem, which was strengthened to hold even if the pieces have to be Borel sets (Marks and Unger). We will revisit this problem focusing on the condition on the boundaries.

On measures that improve L^q dimension under convolution**Eino Rossi***(University of Helsinki)*

The L^q dimension of a probability measure μ , denoted by $L(\mu, q)$, is one way of measuring the smoothness of μ . Heuristically, convolution is a smoothing operation, so L^q dimension should increase in convolutions. We give two different general criteria which guarantee that the L^q dimension strictly increases in convolution. Some classes that satisfy one of the criteria are for example Ahlfors regular measures, measures supported on porous sets, and Moran construction measures. Our results hold for any finite $q > 1$ and thus we also have corollaries about the improvement of the L^∞ dimension, which is the limit of $L(\mu, q)$ as $q \rightarrow \infty$, or equivalently the supremum of the Frostmann exponents of μ .

The dimension results, follow from discrete results about improvement of L^q norms in a given level, and those results in turn are obtained using Shmerkin's inverse theorem for L^q norms. We also discuss applications to sets.

The talk is based on a collaboration with Pablo Shmerkin.

Fubini-type results for Hausdorff dimension**Kornélia Héra***(Eötvös Loránd University, Budapest)*

It is well known that for Hausdorff dimension the naive Fubini theorem does not hold. Namely, there exist sets $E \subset \mathbb{R}^n$ such that for all $x \in \mathbb{R}$, the vertical sections $E_x = \{y \in \mathbb{R}^{n-1} : (x, y) \in E\}$ have Hausdorff dimension s , and $\dim_H E > s + 1$.

We prove a weaker variant of the Fubini theorem for Hausdorff dimension. Namely, for any Borel set B there is a small subset $G \subset B$ (in an appropriate sense) such that for $B \setminus G$ the naive Fubini theorem holds, that is, $\dim_H(B \setminus G) = s + 1$, where s is the essential supremum of the vertical sections $(B \setminus G)_x, x \in \mathbb{R}$. Our results imply that for certain unions of lines, we do not even need to remove a subset for the naive Fubini theorem to be valid.

As a consequence we also obtain projection theorems.

Joint work with Tamás Keleti and András Máthé.

Interval projections of self-similar sets**Abel Farkas***(Alfréd Rényi Institute of Mathematics)*

A 1-dimensional self-similar set on the plane may have infinitely many projections of positive Lebesgue measure. If the open set condition is satisfied then the projection has positive measure if and only if it contains an interval. In this talk we discuss the question when can the projection be an interval. Under quite general conditions we show that only finitely many projection is an interval.

On families of labyrinth fractals**Ligia Loretta Cristea***(Universität Graz, Austria)*

An $n \times n$ *pattern* is obtained by dividing the unit square into $n \times n$ congruent smaller sub-squares and colouring some of them in black (which means that they will be cut out), and the rest in white.

By using special patterns, which we called “labyrinth patterns”, we create and study a special class of Sierpiński carpets, called labyrinth fractals [1, 2]. Labyrinth fractals are dendrites. We study properties of the curves in these dendrites, in particular, their length. Under certain conditions on the patterns we obtain objects with some “magic” properties.

First, we study the self-similar case. Already in this “simplest” case results from several areas of mathematics (topology, combinatorics, linear algebra, curves theory, graph theory) are needed in order to establish the main results. An important role is played here by the path matrix of a pattern or a labyrinth set. We show that under special conditions on the labyrinth patterns, the arcs between any distinct points of the labyrinth fractals have infinite length.

As a next step, we introduce and study mixed labyrinth fractals [3], which are not self-similar. It is interesting to see here which properties are inherited from the self-similar case, and which are not. The results show how by an appropriate choice of the labyrinth patterns, one can obtain . . . almost anything [4].

In very recent research [5] we study an even more general class, called supermixed labyrinth fractals, and solve a conjecture on mixed labyrinth fractals. Every time we pass to a more general class, it is necessary to introduce new objects and tools and to use new techniques for our proofs.

A further generalisation are wild labyrinth fractals . . .

There is ongoing work on a new class of labyrinths, which I also plan to refer to in my talk.

It is worth mentioning that some of our results on labyrinth fractals have already been used by physicists in their research and construction of prototypes. Moreover, we are aware that these objects are suitable as future models for certain crystals, as other recent research in physics shows.

- [1] L.L. Cristea, B. Steinsky, *Curves of Infinite Length in 4×4 -Labyrinth Fractals*, Geometriae Dedicata, Vol. 141, Issue 1 (2009), 1–17
- [2] L.L. Cristea, B. Steinsky, *Curves of Infinite Length in Labyrinth-Fractals*, Proceedings of the Edinburgh Mathematical Society Volume 54, Issue 02 (2011), 329–344
- [3] L.L. Cristea, B. Steinsky, *Mixed labyrinth fractals*, Topol. Appl.(2017), <http://dx.doi.org/10.1016/j.topol2017.06.022>
- [4] L.L. Cristea, G. Leobacher, *On the length of arcs in labyrinth fractals*, Monatshefte für Mathematik (2017), doi:10.1007/s00605-017-1056-8
- [5] L.L. Cristea, G. Leobacher, *Supermixed labyrinth fractals*, submitted for publication (2018)

Parallel Session 1

Saurabh Verma

17:30 – 17:50

A Revisit to Fractal Interpolation Function and a Fractalization of Rational Trigonometric Functions

Saurabh Verma

(Indian Institute of Technology Delhi)

The concept of fractal interpolation function was introduced by Barnsley [Constr. Approx., 2(1986), pp. 303 – 329] on the basis of the theory of iterated function system (IFS). The notion of fractal interpolation provides a bounded linear operator, the so-called fractal operator, which maps a real-valued continuous function on a closed bounded interval in \mathbb{R} to its fractal counterpart with a specified roughness. The first part of the current study is targeted to record continuous dependence of the fractal interpolation function on various parameters involved in the corresponding IFS. In the second part, we explore the idea of fractal operator to provide a new approximation class referred to as fractal rational trigonometric functions. We establish the existence of a best fractal rational trigonometric approximant to a continuous function. Furthermore, we investigate an upper bound for the smallest error in approximating a prescribed continuous function by a fractal rational trigonometric function. This extempore similar results in the setting of fractal rational functions appeared in [J. Approx. Theory, 185(2014), pp. 31 – 50].

Parallel Session 2

Shilei Kong

16:00 – 16:20

Random walks and induced energy forms on compact doubling spaces

Shilei Kong

(Bielefeld University)

A successive partition on a compact doubling space K brings a natural augmented tree structure (X, E) that is Gromov hyperbolic, and the hyperbolic boundary is Hölder equivalent to K . In this talk we introduce a class of transient reversible random walks on (X, E) with return ratio λ . Using Silverstein's theory of Markov chains, we prove that the random walk induces an energy form on K with

$$\mathcal{E}_K[u] \asymp \iint_{K \times K \setminus \Delta} \frac{|u(\xi) - u(\eta)|^2}{V(\xi, \eta) \rho(\xi, \eta)^\beta} d\mu(\xi) d\mu(\eta),$$

where $V(\xi, \eta)$ is the μ -volume of the ball centered at ξ with radius $\rho(\xi, \eta)$, Δ is the diagonal, and β depends on λ . This is a joint work with Ka-Sing Lau and Ting-Kam Leonard Wong.

Parallel Session 2

Jun Kigami

16:20 – 16:40

Weighted partition of a compact metric space, its hyperbolicity and Ahlfors regular conformal dimension

Jun Kigami

(Kyoto University)

Successive divisions of compact metric spaces appear in many different areas of mathematics such as the construction of self-similar sets, Markov partition associated with hyperbolic dynamical systems, dyadic cubes associated with a doubling metric space. The common feature in these is to divide a space into a finite number of subsets, then divide each subset into pieces and repeat this process again and again. In this paper we generalize such successive divisions and call them partitions. Given a partition, we consider the notion of a weight function assigning a “size” to each piece of the partition. Intuitively we believe that a partition and a weight function should provide a “geometry” and an “analysis” on the space of our interest. In this paper, we are going to pursue this idea in three parts. In the first part, the metrizable of a weight function is shown to be equivalent to the Gromov hyperbolicity of the graph associated with the weight function. In the second part, the notions like bi-Lipschitz equivalence, Ahlfors regularity, the volume doubling property and quasisymmetry will be shown to be

equivalent to certain properties of weight functions. In particular, we find that quasisymmetry and the volume doubling property are the same notion in the world of weight functions. In the third part, a characterization of the Ahlfors regular conformal dimension of a compact metric space is given as the critical index p of p -energies associated with the partition and the weight function corresponding to the metric.

Parallel Session 2

Huojun Ruan

16:40 – 17:00

Metrics on the Sierpinski carpet by weight functions

Huojun Ruan
(Zhejiang University)

Let K be the Sierpinski carpet generated by $\{F_i\}_{i=1}^8$, where the fixed points of F_1, F_3, F_5 and F_7 are the four vertices of the square $[0, 1]^2$. Given $a, b \in (0, 1)$, we can define a weight function $g_{a,b} : \Sigma_8^* \rightarrow (0, 1)$ by $g_{a,b}(w) = \prod_{j=1}^n r_{w_j}$ for $w = w_1 \cdots w_n$, where $r_j = a$ if $j = 1, 3, 5, 7$, and $r_j = b$ otherwise. Kigami introduced a pseudo-metric $D_{g_{a,b}}$ on K by

$$D_{g_{a,b}}(x, y) = \inf \left\{ \sum_{i=1}^m g_{a,b}(w(i)) \mid (w(1), \dots, w(m)) \text{ is a chain between } x \text{ and } y \right\},$$

where $(w(1), \dots, w(m))$ is called a chain between x and y if $w(i) \in \Sigma_8^*$ and $K_{w(i)} \cap K_{w(i+1)} \neq \emptyset$ for all i , and $x \in K_{w(1)}$, $y \in K_{w(m)}$. He also conjectured that

$D_{g_{a,b}}$ is a metric if and only if $2a + b \geq 1$ and $a + 2b \geq 1$.

In this talk, we will show that the conjecture holds. Some applications are also discussed. This is a joint work with Qing-Song Gu and Hua Qiu.

Parallel Session 2

Meng Yang

17:10 – 17:30

Local and Non-Local Dirichlet Forms on the Sierpinski Carpet

Meng Yang
(University of Bielefeld)

We give a purely analytic construction of a self-similar local regular Dirichlet form on the Sierpinski carpet using approximation of stable-like non-local closed

forms which gives an answer to an open problem in analysis on fractals.

Parallel Session 2

Elias Hauser

17:30 – 17:50

Oscillations on the Stretched Sierpinski Gasket

Elias Hauser

(University of Stuttgart)

The *Stretched Sierpinski Gasket* (SSG) (or *Hanoi attractor*) is an example of a non-self-similar fractal that still exhibits a lot of symmetry. The existence of various symmetric resistance forms on the SSG was shown in 2016 by Alonso-Ruiz, Freiberg and Kigami. To get self-adjoint operators from these resistance forms we have to choose a locally finite measure. In a previous work we calculated the leading term for the asymptotics of the eigenvalue counting function from these operators. In this talk we would like to refine these results. In particular we want to look for periodic behaviour in the leading term. To answer this question, we prove the existence of localized eigenfunctions and show non-convergence.

Parallel Session 2

Lenon Minorics

17:50 – 18:10

Some Limit Theorems for the Laplacian on Statistically Self-Similar Cantor Strings

Lenon Minorics

(Universität Stuttgart)

We study the spectral asymptotics of some open subsets of the real line with random fractal boundary. Firstly, we establish a strong law of large numbers for the eigenvalue counting function which leads to the second order term in the Weyl asymptotics. Afterwards, we discuss the random fluctuation of the normalized eigenvalue counting function around its limit by giving a central limit theorem. Since the central limit theorem only makes a statement about convergence in distribution, we also establish an almost sure error estimate of the random fluctuation using a law of the iterated logarithm.

Parallel Session 3

Antti Käenmäki

16:00 – 16:20

Domination and thermodynamic formalism for planar matrix cocycles

Antti Käenmäki

(University of Eastern Finland)

We consider cocycles in the simplest non-commutative setting, namely in the case of planar matrices. A cocycle is dominated if there is a uniform exponential gap between singular values of its iterates. This is equivalent to the existence of a strongly invariant multicone in the projective space. We show that a planar matrix cocycle is dominated if and only if matrices are hyperbolic and the norms in the generated sub-semigroup are almost multiplicative.

Matrix cocycles appear naturally in the study of random matrix products and in thermodynamic formalism for matrix-valued potentials. A norm potential satisfying domination is a prime example of an almost-additive dynamical system. We show that all such systems can be studied with the classical thermodynamic formalism. In fact, we are able to characterize all the properties of equilibrium states for norm potentials by means of the properties of matrices. As a consequence of our results, answering a folklore question, we show the existence of a quasi-Bernoulli equilibrium state which is not a Gibbs measure for any Hölder continuous potential.

The talk is based on a recent work with B. Barany and I. D. Morris.

Parallel Session 3

Bing Li

16:20 – 16:40

Metric recurrence and shrinking target theory in dynamical systems

Bing Li

(South China University of Technology)

We consider the recurrence and well-approximable sets for a general dynamical system with mild conditions including β -transformations, continued fraction, piecewise monotone dynamical systems, conformal repeller etc. A dichotomy for their measures is obtained. This is a joint work with Mumtaz Hussain, David Simmons and Baowei Wang.

Parallel Session 3

András Telcs

16:40 – 17:00

Inference of causal relations via dimensions

András Telcs

(MTA Wigner RCP)

Zsigmond Benkő¹, Adám Zlatniczki¹, Lóránd Erőss², Dániel Fabó² András Telcs^{1,4,5}, Zoltán Somogyvári^{1,3}

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Causality is one of the fundamental pillars of science. This paper presents the new Dimensional Causality method which is able to detect and assign probabilities for all types of causal relationships: independence, direct or circular causal connection(s) as well as the existence of hidden common cause. It is based on intrinsic dimension estimates of the joint and separate time delay embedding of the time series which is homeomorphic to the attractor of the underlying systems. We demonstrate the capabilities of our method on simulated examples and (as well as) human neuro-electrophysiological measurements.

The increase of common cause probability during evoked activity of patient's EEG signals by photo stimulation is properly detected. During epileptic seizures the method reveals direct drive from the possible seizure onset zone and found common cause between the signals from the driven areas as well. The new method provides much clearer picture of the interactions between recorded time series and promises applicability in many branches of science.

Friday, 5 October 2018

Chair: Kigami

9:00 – 9:50 **KN Bonk**

The quasiconformal geometry of continuum trees

9:50 – 10:25 **IT Kajino**

The Laplacian on some round Sierpiński carpets and Weyl's asymptotics for its eigenvalues

coffee break

11:00 – 11:35 **IT Teuf**

Loops in Sierpinski graphs

11:35 – 12:10 **IT Keleti**

Recent progress on the dimensions of planar distance sets I

12:10 – 12:45 **IT Shmerkin**

Recent progress on the dimensions of planar distance sets II

13:00 – 14:00 lunch

KN

Mario Bonk

09:00 – 09:50

The quasiconformal geometry of continuum trees

Mario Bonk

(University of California Los Angeles)

Continuum trees appear in various contexts: in probabilistic models, as Julia sets of polynomials, or as attractors of iterated function systems. Accordingly, one wants to gain a good understanding of the topology and geometry of these objects, but often faces difficult problems. For example, it was not known until recently whether two independent samples of the CRT (continuum random tree) are almost surely homeomorphic. Even more difficult questions arise if one investigates the quasiconformal geometry of continuum trees, and more specifically, if one wants to characterize a given tree up to quasisymmetric equivalence. In my talk I will present some recent developments in this area. My talk is based on joint work with Huy Tran (TU Berlin) and with Daniel Meyer (U. Liverpool).

IT

Naotaka Kajino

09:50 – 10:25

The Laplacian on some round Sierpiński carpets and Weyl's asymptotics for its eigenvalues

Naotaka Kajino

(Kobe University)

The purpose of this talk is to present the speaker's recent result on the construction of a "canonical" Laplacian on round Sierpiński carpets invariant with respect to certain Kleinian groups (i.e., discrete groups of Möbius transformations on $\widehat{\mathbb{C}} := \mathbb{C} \cup \{\infty\}$) and on Weyl's asymptotics for its eigenvalues. Here a *round Sierpiński carpet* refers to a subset of $\widehat{\mathbb{C}}$ homeomorphic to the standard Sierpiński carpet, such that its complement in $\widehat{\mathbb{C}}$ consists of disjoint open disks in $\widehat{\mathbb{C}}$.

The construction of the Laplacian is based on the speaker's preceding study of the simplest case of the *Apollonian gasket*, the compact fractal subset of \mathbb{C} obtained from an ideal triangle (a triangle formed by mutually tangent three circles) by repeating indefinitely the process of removing the interior of the inner tangent circles of the ideal triangles. On this fractal, Teplyaev (2004) had constructed a canonical Dirichlet form as one with respect to which the coordinate functions on the gasket are harmonic, and the author later proved its uniqueness and discovered an explicit expression of it in terms of the circle packing structure of the gasket.

The expression of the Dirichlet form obtained for the Apollonian gasket in fact makes sense on general circle packing fractals, including round Sierpiński carpets, and defines (a candidate of) a “canonical” Laplacian on such fractals. When the circle packing fractal is the limit set (i.e., the minimum invariant non-empty compact set) of a certain class of Kleinian groups, some explicit combinatorial structure of the fractal is known and makes it possible to prove Weyl’s asymptotic formula for the eigenvalues of this Laplacian, which is of the same form as the circle-counting asymptotic formula by Oh and Shah [Invent. Math. **187** (2012), 1–35].

The overall structure of the proof of Weyl’s asymptotic formula is the same as in the case of the Apollonian gasket and is based on a serious application of Kesten’s renewal theorem [Ann. Probab. **2** (1974), 355–386] to a certain Markov chain on the “*space of all possible Euclidean shapes*” of the fractal. There is, however, a crucial difficulty in the case of a round Sierpiński carpet; since it is *infinitely ramified*, i.e., the cells in its cellular decomposition intersect on infinite sets, it is highly non-trivial to show that the principal order term of the eigenvalue asymptotics is not affected by the cellular decomposition, namely by assigning the Dirichlet boundary condition on the boundary of the cells. This is achieved by utilizing (1) an upper bound on the heat kernel obtained from a version of the Nash inequality, and (2) the geometric property, noted by M. Bonk in [Invent. Math. **186** (2011), 559–665], that the circles $\{C_k\}_{k=1}^\infty$ in the round carpet are *uniformly relatively separated*: there exists $\delta \in (0, \infty)$ such that

$$\text{dist}(C_j, C_k) \geq \delta \min\{\text{rad}(C_j), \text{rad}(C_k)\} \quad \text{for any } j, k \geq 1 \text{ with } j \neq k.$$

IT

Elmar Teufl

11:00 – 11:35

Loops in Sierpinski graphs

Elmar Teufl

(University of Tübingen)

We introduce two random loop models on Sierpinski graphs. In the first model the edge set is partitioned randomly into cycles (random edge cycle cover). In the second model the vertex set is partitioned randomly into cycles (random 2-factor). Some results on the asymptotics of short and long cycles are discussed. The long cycles in these two models have a quite different behaviour.

IT

Tamás Keleti

11:35 – 12:10

Recent progress on the dimensions of planar distance sets I**Tamás Keleti***(Eötvös Lorand University, Budapest)*

We will discuss our recent work on the dimensions of the distance sets of planar Borel sets. Among other results, we establish (the dimension version of) Falconer's distance set conjecture for a large class of planar sets, and obtain improved bounds for the (Hausdorff and packing) dimensions of the distance sets of Borel sets of dimension larger than 1.

IT

Pablo Shmerkin

12:10 – 12:45

**Recent progress on the dimensions of planar distance sets
II****Pablo Shmerkin***(Torcuato Di Tella University, Buenos Aires)*

We will discuss our recent work on the dimensions of the distance sets of planar Borel sets. Among other results, we establish (the dimension version of) Falconer's distance set conjecture for a large class of planar sets, and obtain improved bounds for the (Hausdorff and packing) dimensions of the distance sets of Borel sets of dimension larger than 1.

Poster Contributions

Posters are displayed for the duration of the conference, they are also available online at

<http://fgs6.math.kit.edu/72.php>

There is a dedicated **Poster Session** on Tuesday evening from 20:00 to 21:30 h.

Please take note of the **Best Poster Award** sponsored by the Birkhäuser publishing house. Your conference booklet contains a ballot slip for your choice of the three best posters. Please hand in your completed ballot slip by the coffee break on Wednesday morning. The ballot box will be placed in the main lecture hall.

Poster 1

Catherine Bruce

Projections of Gibbs measures on self-conformal sets**Catherine Bruce***(University of Manchester)*

Hochman and Shmerkin used Furstenberg's theory of CP-processes to prove strong Marstrand results for self-similar sets and measures with dense rotations which satisfy the strong separation condition. That is, to prove that the Hausdorff dimension of the projections of such sets and measures is the maximum possible value for every projection. Here we extend such a result to Gibbs measures on self-conformal sets without requiring any separation condition. The extension relies on some careful estimates of the entropy growth of non-linearly zoomed-in measures and the distortion of conformal iterated function systems under orthogonal projections. The result applies to Gibbs measures on hyperbolic Julia sets.

Poster 2

Stuart Burrell

The dimension of inhomogeneous self-affine sets**Stuart Burrell***(University of St Andrews)*

The dimension of inhomogeneous attractors, introduced independently by Barnsley (1985) and Hata (1985), exhibit interesting behaviour. Of particular interest, is asking how the dimension of the inhomogeneous attractor is related to the corresponding dimensions of the homogeneous attractor and the condensation set. For iterated functions consisting of similarity mappings, this question was answered by Fraser (2012). In 2018, we generalised this to arbitrary iterated function systems by introducing the notion of upper Lipschitz dimension. This also answered the case for self-affine inhomogeneous sets with affinity dimension less than one. In this talk we'll cover recent results satisfying the case for arbitrary inhomogeneous self-affine sets of any dimension.

Random Cantor sets with dependence

Wafa Chaouch Ben Saad
(*University of Bremen*)

Consider a random Cantor set that is generated by a binary tree-indexed family of random contractions. Without imposing the independence of the contractions we determine an almost sure upper bound of its Hausdorff dimension in terms of random pressure functions. Following the work of K. Falconer, B. M. Hambly and others.

Relative entropy of chaotic

Mauricio Díaz
(*UNAB*)

In this article we research about the chaotic system seeing for Glasnert and Weiss and show that exist a non algebraic relation of sets equivalent to a χ -system and P-system and then the topological entropy relative to a partition of the sum of both system is zero for non trivial solutions of a measure μ . After that we proof that a chaotic system can be studied using a stable set for a periodic point p and also for any operation with an unstable set if the map has inverse on the set X.

Wavelet analysis of a multifractional process in an arbitrary Wiener chaos

Yassine Esmili
(*Université de Lille*)

The well-known multifractional Brownian motion (mBm) is the paradigmatic example of a continuous Gaussian process with non-stationary increments whose local regularity changes from point to point. In this work, using a wavelet approach, we construct a natural extension of mBm which belongs to a homogeneous Wiener chaos of an arbitrary order. Then, we study its global and local behavior.

Applying Saari triangles in the contexts of the Koch Snowflake: voting procedures embedded in prefractal examples

Jaakko Hakula

(Freelance researcher)

The idea to combine works of the two eminent mathematicians - von Koch (with Koch genealogies) and Saari - was initiated by a haphazard intuition when searching possible ways to utilize equilateral triangles in visualizations of MCDM (multi-criteria decision-making). The aim of the study is to apply some basic ideas of positional voting theory in the case of three alternatives depicting the context with Saari triangles as the initiators of the triadic Koch curve, i.e. the Koch snowflake. The crux of the application is to choose the alternatives to fit “metrically” in the determinants of the Koch snowflake. The three alternatives A, B and C most convenient to be chosen would be either the indentation angle or in the vector-based solution the direction of the generative vector. The former was chosen as a more simple and still sufficiently informative one compared to the latter. The three alternatives (i.e. angles) stand for 10, 30 and the conventional 60 degrees. The positional voting rules applied are the plurality, the Borda and the antiplurality rules. For the three alternatives there are $3! = 6$ voter types. A profile specifies the number of voters of each type. With the same amount and division of votes given to each profile, different methods do not necessarily give the same outcomes. Further on, the “best” indentation angle tallied is utilized to create variants of the generalized Koch curve. The changes in the indentation angle make the scaling factor defining the self-similarity dimension to be a function of the indentation angle. Alternating the scope of the indentation angle ends up with geometries of various fractal dimensions in the generalized triadic Koch curves. In conclusion, experimenting with the central conditions of the aforementioned parameters a number of prefractal images of the Koch snowflakes are generated. Further studies will concentrate on searching possibilities of multifractal properties in the settings of the Saari-Koch hybrids.

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Poster 7

Kirill Kamalutdinov

Even one intersection point can crash OSC: an example

Kirill Kamalutdinov
(*Novosibirsk State University*)

The violation of the open set condition (OSC) is caused by overlaps of a self-similar set, generated by system \mathcal{S} of similarities. If OSC does not hold, there is at least one point in critical set of the system \mathcal{S} , but there is no guarantee that this point is unique. Using General Position Theorem, we construct a family of self-similar sets in $[0, 1]$ generated by system \mathcal{S}_{pqr} of six similarities depending on a parameters (p, q, r) , such that for Lebesgue-almost all (p, q, r) in some parameter space, a critical set of the system \mathcal{S}_{pqr} consist of exactly one point, but the system \mathcal{S}_{pqr} does not have weak separation property, so it does not satisfy OSC.

This is joint work with Andrei Tetenov.

This work was supported by the Laboratory of Topology and Dynamics, Novosibirsk State University (contract no. 14.Y26.31.0025 with the Ministry of Education and Science of the Russian Federation).

Poster 8

Stefan Kohl

Martin boundary theory on the weighted Sierpinski gasket**Stefan Kohl***(University of Stuttgart)*

We want to extend the Martin boundary theory on fractals, which are generated by probabilistic iterated function systems. In this case, each contraction gets an additional probability, also called weight. For simplicity we investigate this new idea on the weighted Sierpinski gasket, where all weights are positive, but may be different. The weights force us to rethink the definition of the probabilities moving from a parent cell to a children cell. This leads to the consequence, that the green function changes and also the Martin kernel is different from the one in the unweighted case. As we will see, the unweighted case can also be characterized within this new ansatz as a special case. Since this is work in progress, further results may be received till the conference.

Poster 9

Melissa Meinert

Sobolev spaces and calculus of variations on fractals**Melissa Meinert***(Bielefeld University)*

We consider Sobolev spaces on metric measure spaces that carry a strongly local regular Dirichlet form. Our aim is to generalize some basic results from the calculus of variations, such as the existence of minimizers for convex functionals, with the help of these Sobolev spaces. This applies to a number of non-classical situations such as degenerate diffusions, superpositions of diffusions and diffusions on fractals or on products of fractals. Based on joint work with Michael Hinz and Dorina Koch.

Poster 10

Christina Moor

Conformal function systems with weak separation condition**Christina Moor***(University of Bremen)*

In the context of conformal iterated function systems and conformal graph directed systems, we will study the analogue phenomenon of dimension drop from

symbolic dimension to Hausdorff dimension. In particular, building on ideas of Lau, Ngai and Wang, we will investigate the condition of weak separation for conformal systems.

Poster 11

Mariusz Olszewski

Good labelling property of simple nested fractals

Mariusz Olszewski

(Wrocław University of Science and Technology)

In the paper “The Lifschitz singularity for the density of states on the Sierpinski gasket” K. Pietruska-Pałuba constructed a reflected Brownian motion on the Sierpiński gasket. A crucial element of the construction was the labelling of the fractal vertices which allowed to define folding projections on a complex of a given size.

A similar approach adapted to simple nested fractals leads to what is called the good labelling property (GLP in short). On fractals satisfying GLP one can define the reflected Brownian motion using similar techniques as on Sierpiński gasket.

The Lindstrom snowflake is the basic example showing that the GLP is indeed mandatory for construction of the reflected Brownian motion.

We give detailed geometrical conditions implying the GLP on planar fractals and show that the class of fractals satisfying the GLP is very rich. We also present the properties of projections based on the GLP.

Poster 12

Srijanani Anurag Prasad

Multiresolution analysis based on Coalescence Hidden-variable Fractal Interpolation Functions.

Srijanani Anurag Prasad

(Indian Institute of Technology Tirupati)

In this presentation, Multi resolution analysis arising from Coalescence Hidden-variable Fractal Interpolation Functions (CHFIFs) is developed. The availability of a larger set of free variables and constrained variables with CHFIF in multi resolution analysis based on CHFIFs provides more control in reconstruction of functions than that provided by multiresolution analysis based only on Affine Fractal Interpolation Functions (AFIFs). In this presentation, I shall first introduce the vector space of CHFIFs, determine the dimension of that vector space and then construct Riesz bases of vector sub-spaces consisting of certain functions which are CHFIFs.

Poster 13

Mounika Rapolu

Multifractal, Fractal and Lacunarity analysis of the three-dimensional cerebral vasculature of the mouse brain in vivo

Mounika Rapolu*(Institute of Physical Chemistry, Polish Academy of Sciences)*

In this paper, we present the application and analysis of multi fractal, fractal and lacunarity for the in vivo studies of cerebral region of the healthy mice brain obtained from experimental studies using optical coherence microscopy(OCM). These parameters are also known to be the hallmarks for the analysis of characteristic tumor angiogenesis, metastasis and invasion. But the studies in literature still lack for the conjecture for smaller vessels of the resolution $\sim 2\mu m$, which are the strong tool (building blocks) for the detailed understanding of small tumor growth and it deserves a more detailed analysis. The goal would be to discuss the merits at this resolution in detail and to obtain the thresholding values for further tracking of the dynamic change of these parameters from transition of healthy micro vessels to glioblastoma tumors vessels.

Poster 14

Natalia Saburova

Spectrum of Laplacians on periodic graphs with guides

Natalia Saburova*(Northern (Arctic) Federal University)*

We consider Laplace operators on periodic discrete graphs perturbed by guides, i.e., graphs which are periodic in some directions and finite in other ones. The spectrum of the Laplacian on the unperturbed graph is a union of a finite number of non-degenerate bands and eigenvalues of infinite multiplicity. We show that the spectrum of the perturbed Laplacian consists of the unperturbed one plus the additional so-called guided spectrum which is a union of a finite number of bands. We estimate the position of the guided bands and their length in terms of geometric parameters of the graph. We also determine the asymptotics of the guided bands for guides with large multiplicity of edges. Moreover, we show that the possible number of guided bands, their length and position can be rather arbitrary for some specific periodic graphs with guides. This is a joint work with Evgeny Korotyaev from St. Petersburg State University.

Poster 15

Abhilash Sahu

On the box dimension of graph of harmonic function on the Sierpiński gasket

Abhilash Sahu

(Indian Institute of Technology Delhi)

In this talk we will give bounds for the box dimension of graph of harmonic function on the Sierpiński gasket. Also we get an upper and a lower bound for the box dimension of graph of functions that belongs to $\text{dom}(\mathcal{E})$, that is, all finite energy functionals on the Sierpiński gasket. Also, we show the existence of fractal functions in the function space $\text{dom}(\mathcal{E})$ with the help of fractal interpolation functions. Further we give bounds for some functions which belongs to the family of continuous functions and arises as Fractal interpolation function.

Poster 16

Karenina Sender

Martin boundary and minimal Martin boundary for Markov chains

Karenina Sender

(Universität Bremen)

The notion of Martin boundary for Markov chains was first introduced by Doob (1959) and Hunt (1960). Here, constructions of Markov chains with fractal Martin boundary, especially the Sierpiński gasket, are given.

Poster 17

Shuang Shen

Extended multifractal formalism of some non-doubling measures

Shuang Shen

(Northwestern Polytechnical University)

We constructed in a previous work measures on symbolic spaces which satisfy an extended multifractal formalism, in the sense that Olsen's functions b and B differ and that their Legendre transforms have the expected interpretation in terms of dimensions. These measures are composed with a Gray code and projected onto the unit interval so to get doubling measures. We were able to show that the projected measure has the same Olsen's functions as the one it

comes from and that it also fulfills the extended multifractal formalism. Here we show that the use of the Gray code is not necessary to obtain these results, although dealing with non-doubling measures.

Poster 18**Jan Simmer**

Spectral and resolvent convergence for magnetic Laplacians on finitely ramified sets

Jan Simmer
(Universität Trier)

We present some recent results on norm resolvent and spectral convergence for a compatible sequence (i.e., a sequence of discrete magnetic Laplacians) approximating a finitely ramified set with magnetic Laplacian.

Poster 19**Klemens Taglieber**

Walk dimension of Vicsek sets with different scaling factors

Klemens Taglieber
(University of Stuttgart)

In this talk we present a method for calculating the walk dimension which was introduced by Grigor'yan and Meng to determine the walk dimension of the Sierpiński gasket.

We apply this method to the Vicsek set with scaling factor 3^{-1} by constructing a graph sequence which converges to the fractal. On these pregraphs we define Dirichlet forms and continue them on the Vicsek set. The critical parameter for the triviality of the Dirichlet space yields the walk dimension. We further investigate Vicsek sets with scaling factors $(2n+1)^{-1}$, $n \in \mathbb{N}$ and take a look at the limit of the walk dimensions as $n \rightarrow \infty$.

Singular measures for random piecewise affine interval homeomorphisms

Adam Śpiewak
(*University of Warsaw*)

We consider a random dynamical system consisting of two piecewise affine increasing homeomorphisms f_1, f_2 of the unit interval, iterated randomly with probabilities (p_1, p_2) . If the system satisfies $f_1(x) < x < f_2(x)$ for $x \in (0, 1)$ and both endpoint Lyuapunov exponents $\sum_{i=1}^2 p_i \ln f'_i(0)$, $\sum_{i=1}^2 p_i \ln f_i(1)$ are positive, then the system admits a unique stationary probability measure μ with no atoms at the endpoints. In this case, μ has to be either singular or absolutely continuous. We prove that μ is singular for systems satisfying certain symmetry and resonance conditions, verifying a conjecture by Alsedá and Misiurewicz in this case. We also calculate Hausdorff dimensions of the measure μ and its support. This is joint work with Krzysztof Barański.

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